Quasi-stationary AC Analysis Using Phasor Description With Modelica

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Abstract

The investigation of large technical systems by simulating long time periods requires very effective methods. If only sinusoidal quantities occur in the electrical domain, phasor analysis can be used to describe the steady-state behaviour of this part of the physical system. In this paper, modelling of AC circuits and electromechanical drives with electrical AC subsystems is presented using the well-known phasor method within Modelica. To this end, some fundamentals concerning phasor description are repeated before a possible implementation of AC circuits within Modelica is proposed. This implementation uses a new Modelica-library which is still under construction. The main content of this library is introduced. Furthermore, some statements are given concerning the library's usage when coupling with other domains or with transient submodels, or when switching between transient and phasor analysis, respectively. Finally, three examples are presented.
Keywords: sinusoidal quantity, steady-state analysis, phasor domain models, model coupling, variable model structure

1 Introduction

Many linear electrical circuits use alternating current (AC circuits). Most of them operate with a nominal frequency and nearly ideal sinusoidal electrical quantities. Distortions occuring due to circuits' nonlinearities can often be neglected. This way, we can define so called *idealized linear AC circuits*. They shall be characterized by ideal sinusoidal quantities and one single (nominal) frequency.

Three operating modes can be distinguished in idealized linear AC circuits: the steady-state mode, the dynamic mode, and the so-called *quasi-stationary* mode. The first mode is characterized by constant amplitudes and phases of all sinusoidal quantities. The system yields the sinusoidal steady-state response. During the second mode, „fast" dynamic changes of sinusoidal quantities occur. „Fast" means that the appearing dynamic processes shall have a low dominant time constant compared to the nominal frequency. Usually, it is a good choise if this time constant is less than $10T$ ($T = 1/f$, f – nominal frequency). In this case, the full transient (or complete) response of a system has to be considered (using e.g. the electrical Modelica standard library [14] or Haumer's libraries [6]). Such „fast" dynamic processes appear with switching operations or (stepwise or „fast" continuous) changes of parameters. Because consisting only of decreasing shares in most cases, they fade away with advancing time and can be neglected after a finite time period. The third mode – the so-called *quasi-stationary* mode – shall be understood as a sequence of steady states on the following condition: parameters (which would be constant at steady-state analysis) may vary extraordinary slowly compared to the nominal frequency. It is signalized by „slow" alterations of amplitudes and phases of the sinusoidal quantities. Usually, it is a good choise if the dynamic processes have a dominant time constant higher than $10T$.

All three operating modes can be investigated by implementing behavioural models (differential-algebraic equations) within appropriate numerical simulation systems. Because the instantaneous values of the sinusoidal quantities are changing perpetually, the performance of dynamic simulations depends on the nominal frequency and, hence, is limited. Especially, the study of such systems for a long time period (hours, days, years) is hardly possible.

In this paper, a very efficient method for modelling AC circuits and its implementation within Modelica is presented. This method can naturally be used for steady-state analysis of such systems. The method's principal idea is the substitution of the sinusoidal (time-depending) physical quantities in the transient model by constant complex quantities – so-called phasors – in the steady-state model. Hence, the system's behaviour is not described by differential but by algebraic equations. The models are called phasor-domain models. This method of steady-state analysis was introduced by Steinmetz in the late 19th century ([10], [11]). Nowadays it is well-known and can be found in many elementary textbooks (e.g. [1], [3], [4], [5], [7], [8], [9]). Wiesmann also uses such an approach in parts of his power-systems library for Modelica ([12]).

Sometimes it is necessary to study physical systems containing both AC circuits and subsystems from other domains for long time periods. Then the quasi-stationary operating mode of AC circuits is of special interest. Considering this mode, a combination of phasor-domain and transient models is possible.

2 Phasor description of AC circuits

2.1 Fundamentals

A sinusoidal signal of the form

$$x(t) = \hat{x}\sin(\omega t + \varphi) \qquad (1)$$

($\hat{x}$ – amplitude, ω – angular frequency, φ – phase) can be understood as a well-defined part of a time-dependent complex quantity

$$\begin{aligned} \underline{x}(t) &= \hat{x}e^{j(\omega t + \varphi)} \\ &= \hat{x}[\cos(\omega t + \varphi) + j\sin(\omega t + \varphi)] \,. \end{aligned} \qquad (2)$$

In the complex plane, signal $\underline{x}(t)$ from (2) describes a rotating vector having a constant length $\hat{x}$ and forming an angle of $(\omega t + \varphi)$ with the real axis at time instant t (see **Fig. 1**). The original signal $x(t)$ can then be gained at any time instant by projecting $\underline{x}(t)$ to the imaginary axis according to

$$\begin{aligned} x(t) &= \mathrm{Im}[\underline{x}(t)] = \mathrm{Im}[\hat{x}e^{j(\omega t + \varphi)}] \\ &= \hat{x}\sin(\omega t + \varphi) \,. \end{aligned} \qquad (3)$$

If the angular frequency ω is constant then the angular velocity of a rotating vector is also constant and, therefore, is not of special interest. The complex quantity $\underline{x}(t)$ and, hence, the original signal $x(t)$ are adequate determined by two values: amplitude $\hat{x}$ and phase φ. Using the rms value $X = \hat{x}/(\sqrt{2})$ of the signal instead of its amplitude, $x(t)$ can be represented by the following phasor

$$\underline{X} = Xe^{j\varphi} \,. \qquad (4)$$

This phasor has the constant length X and forms a constant angle φ with the real axis (see **Fig. 2**).

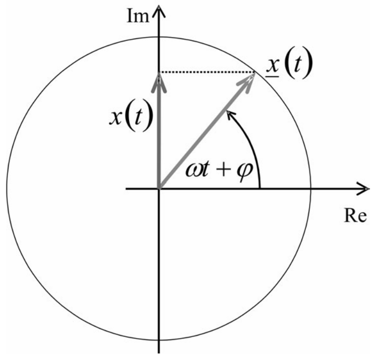

Figure 1: Rotating vector in complex plane

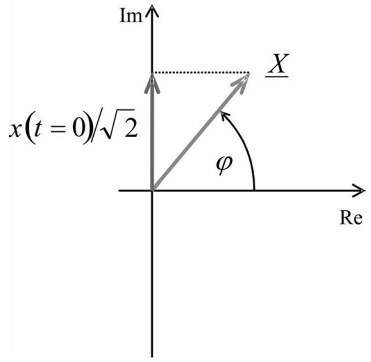

Figure 2: Phasor in complex plane

Please note, that

$$\frac{d}{dt}\{\mathrm{Im}[\underline{X}e^{j\omega t}]\} = \mathrm{Im}[j\omega\underline{X}e^{j\omega t}] \qquad (5)$$

which means that a derivation of $x(t)$ in time domain has to be replaced by multiplication with $j\omega$ in phasor domain.

2.2 Ohm's Law in generalized form

Ohm's Law is well-known in DC analysis. Using phasor description, this law can be written in an analogous form for AC analysis. Ohm's Law reads in its generalized (or complex) form

$$\underline{V} = \underline{Z}\underline{I} \,, \qquad (6)$$

where $\underline{V}$ and $\underline{I}$ are the voltage and current phasors, respectively, and $\underline{Z}$ denotes the impedance or „complex resistance". It holds for linear electric components like resistors, inductors, and capacitors. Consideration of the relations between current and voltage at each type of component yields the corresponding impedances. Assuming a sinusoidal current $i(t) = \hat{i}\sin(\omega t + \varphi_i)$ and a sinusoidal voltage

$v(t) = \hat{v}\sin(\omega t + \varphi_v)$ and replacing them by corresponding phasors $\underline{V}$ and $\underline{I}$, the impedances of each type of component can finally be represented with (5) as:

$$\underline{Z}_R = R, \tag{7}$$

$$\underline{Z}_L = j\omega L, \tag{8}$$

$$\underline{Z}_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C} \tag{9}$$

(indices *R*, *L*, and *C* stand for resistor, inductor, and capacitor, respectively).

In AC circuits, inductors coupled by magnetic fields often occur. The behaviour of such a coupling can easily be included into the phasor description. Let *M* denote the mutual inductance between two coils. Then the EMF in coil 1 caused by a current through coil 2 reads

$$e_1(t) = \mp M\frac{di_2(t)}{dt} \tag{10}$$

(the upper sign holds if the coils are oriented likewise). This yields the following voltage drop

$$v_1(t) = \pm M\frac{di_2(t)}{dt}. \tag{11}$$

With (5), the relations between the current phasor in one branch and the voltage phasor in the other branch read finally

$$\begin{aligned}\underline{V}_1 &= \pm j\omega M\,\underline{I}_2,\\ \underline{V}_2 &= \pm j\omega M\,\underline{I}_1.\end{aligned} \tag{12}$$

2.3 Kirchhoff's Laws

According to Kirchhoff's Current Law, the sum of the instantaneous values of all currents in a node must vanish at each point in time

$$\sum_k i_k(t) = 0, \tag{13}$$

where *k* represents each branch being incident with the considered node. In case of sinusoidal quantities, each current $i_k(t) = \hat{i}_k\sin(\omega t + \varphi_{ik})$ can be determined by projecting a rotating vector

$$\underline{i}_k = \hat{i}_k e^{j(\omega t + \varphi_{ik})} = \sqrt{2}\underline{I}_k e^{j\omega t} \tag{14}$$

to the imaginary axis. That's why it follows from (13)

$$\sum_k \mathrm{Im}[\sqrt{2}\underline{I}_k e^{j\omega t}] = 0 \tag{15}$$

for each time instant *t* which means finally

$$\sum_k \mathrm{Im}[\underline{I}_k] = \mathrm{Im}\Big[\sum_k \underline{I}_k\Big] = 0. \tag{16}$$

Displacing the time axis by $\pi/2$, the currents read $i_k(t) = \hat{i}_k\cos(\omega t + \varphi_{ik})$. Those can be represented by projecting the rotating vector $\underline{i}_k$ to the real axis. Therefore, it holds

$$\mathrm{Re}\Big[\sum_k \underline{I}_k\Big] = 0. \tag{17}$$

Equations (16) and (17) yield finally the generalized form of Kirchhoff's Current Law

$$\mathrm{Re}\Big[\sum_k \underline{I}_k\Big] + j\mathrm{Im}\Big[\sum_k \underline{I}_k\Big] = \sum_k \underline{I}_k = 0. \tag{18}$$

Kirchhoff's Voltage Law says that the sum of the instantaneous values of all voltage drops in one mesh must vanish at each point in time

$$\sum_k v_k(t) = 0. \tag{19}$$

The generalized form of this law can be derived in the very same way as shown above. It reads

$$\sum_k \underline{V}_k = 0. \tag{20}$$

2.4 Coupling phasor domain models and transient models – electric machines

Sometimes, investigations of models consisting of a fast part and a comparatively slow part shall be carried out. If only sinusoidal quantities occur in the fast part of such a model then it can be of interest to use a phasor domain description for this submodel. In these cases, it is necessary to couple at least two submodels: one submodel in phasor-domain description and one transient submodel.

For convenience in the following, phasor-domain models are referred to as *P-models* while transient models are called *T-models*. To couple a P- and a T-submodel within one mathematical description (see **Fig. 3**), some assumptions must be fulfilled:

- the connection between the two submodels consists of one-directional signals only (signals computed within the P-submodel and needed to be known in the T-submodel are referred to as I-signals – input to the phasor domain; signals resulting from the T-submodel and influencing the P-submodel are referred to as O-signals –

output from phasor domain),
- all I-signals are only allowed to alter very slowly compared with the P-submodel's nominal frequency.

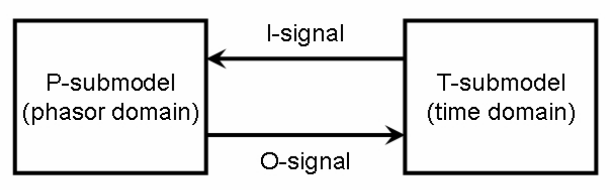

Figure 3: Coupling a P- and a T-submodel

The second point in the list requires either some a priori knowledge about the T-submodel or a permanent check of the first time derivatives of all I-signals. The consideration of all assumptions ensures that the P-submodel is always in steady-state or quasi-stationary mode.

Electric machines consist of a mechanical and an electrical subsystem. The complete response of the mechanical part is very often slow compared to that of the electrical part. This is particularly true with AC machines because of the nominal frequencies mostly used. Under certain assumptions (perfect symmetry, no saturation etc.), the steady state of an AC machine is characterized by ideal sinusoidal voltages and currents with constant amplitudes and phases. The steady state is furthermore signalized by a constant angular velocity of the rotor – with an induction machine – or by a constant torque angle – with a synchronous machine – as well as a constant torque produced electrically with both machines (see e.g. [13]). If the mechanical angular velocity or the torque angle alter very slowly compared to the electrical nominal frequency then the electrical subsystem is in quasi-stationary mode. This results in a slow variance of amplitude and phase of the electric sinusoidal quantities. To study electric machines in steady-state or quasi-stationary mode of the electrical subsystem, a combination of two submodels – a phasor-domain model of the electrical subsystem and a transient model of the mechanical subsystem – is possible.

2.5 Switching between phasor-domain and time-domain models

An electrical model which is to be switched between time domain and phasor domain may be part of a heterogeneous system containing also subsystems of other domains. Therefore, this model is refered to as AC submodel in the following.

If the transient response of an AC subsystem has been faded away then a changeover from time domain to phasor domain is possible because the system is in a quasi-stationary mode or in a steady state. Both states are characterized by constant or at least by nearly constant amplitudes and phases of voltages and currents. During a numeric simulation, such a changeover causes a transition of the formerly differential-algebraic equation system (DAE system describing the AC submodel in time domain) into a linear system of algebraic equations (AE system describing the AC submodel in phasor domain)

$$Ax = b. \tag{21}$$

In (21), matrix A may depend on time t or on state variables of other domains (see e.g. [2]) and, hence, is known at switching time. The elements of vector b either are known (if depending on time or on state variables of other domains) or have to be determined during the transient simulation by "scanning" amplitude and phase continuously. This way, a consistent changeover can be carried out.

A changeover from phasor domain to time domain is always possible. It is actually necessary, if the transient response of an AC subsystem is of interest for a time interval (e.g. caused by a step-wise change of a parameter). In this case, the quasi-stationary or steady state is finished at switching time. During a numeric simulation, the AE system has to be replaced by the corresponding DAE system. Amplitudes and phases of all "transient" source components can be determined from the corresponding phasors. Additionally, initial values for all state variables of the DAE system are needed. These values can be calculated from the voltage phasors across capacitors and from the current phasors through inductors.

3 Implementation in Modelica

3.1 Basic partial models

A new library called `Complex` has been created for the implementation of phasor-domain models in Modelica. This library is designed such that it can be used like the Modelica standard library `Modelica.Electrical.Analog`. An important difference is the definition of a so-called *complex pin* instead of the standard pin to be used for connecting components. Without any annotations, the definition of the complex pin reads

```
connector ComplexPin
  Real vRe, vIm;
  flow Real iRe, iIm;
end ComplexPin;
```

containing real and imaginary part (vRe, vIm) of a voltage phasor $\underline{V}$ as well as real and imaginary part (iRe, iIm) of a current phasor $\underline{I}$. Using this connector, important partial models like OnePort were derived which, first, realize Kirchhoff's relations in generalized form between the component's pins and, second, compute real and reactive (idle) power out of internal quantities and make them available via output signals. The main part of the code for OnePort reads

```
partial model OnePort
  PosComplexPin p;
  NegComplexPin n;
  Real vRe, vIm, iRe, iIm;
  Real phi_v, phi_i, phi;
  Real activePwr, idlePwr;
  Complex.Interfaces.ComplexOutput
    complexPwr;
equation
  vRe = p.vRe - n.vRe;
  vIm = p.vIm - n.vIm;
  iRe = p.iRe;
  iIm = p.iIm;
  0 = p.iRe + n.iRe;
  0 = p.iIm + n.iIm;
  phi_v = Complex.Math.atan2(vIm,vRe);
  phi_i = Complex.Math.atan2(iIm,iRe);
  phi = phi_v-phi_i;
  activePwr = v*i*cos(phi);
  idlePwr = v*i*sin(phi);
  complexPwr.real = activePwr;
  complexPwr.im = idlePwr;
end OnePort;
```

Based on OnePort and other partial models, source components (different voltage sources, some current generators) as well as many linear RLC components for single-phase grids have been created.

Many AC circuits are three-phase systems. To simplify modelling of such systems using the library Complex, the *complex plug* – a special "multi-phase complex pin" – was implemented:

```
connector ComplexPlug
  parameter Integer m(final min=1) = 3;
  Complex.SinglePhase.Interfaces.ComplexPin
    complexpin[m];
end ComplexPlug;
```

Using this connector, some partial models were derived which are suitable to be extended to source components or RLC components of symmetric multi-phase systems. An example of such a partial model is TwoPlug:

```
partial model TwoPlug
  PosComplexPlug plug_p(final m=m);
  NegComplexPlug plug_n(final m=m);
  Real vRe[m], vIm[m], iRe[m], iIm[m];
  Real phi_v[m], phi_i[m], phi[m];
  Real activePwr[m], idlePwr[m];
  Complex.Interfaces.ComplexOutput
    complexPwr[m];
equation
  vRe = plug_p.complexpin.vRe -
        plug_n.complexpin.vRe;
  vIm = plug_p.complexpin.vIm -
        plug_n.complexpin.vIm;
  iRe = plug_p.complexpin.iRe;
  iIm = plug_p.complexpin.iIm;
  for j in 1:m loop
    phi_v[j] =
      Complex.Math.atan2(vIm[j],vRe[j]);
    phi_i[j] =
      Complex.Math.atan2(iIm[j],iRe[j]);
    activePwr[j] = v[j]*i[j]*cos(phi[j]);
    idlePwr[j] = v[j]*i[j]*sin(phi[j]);
  end for;
  phi = phi_v-phi_i;
  complexPwr.real = activePwr;
  complexPwr.im = idlePwr;
end TwoPlug;
```

This model mainly realizes Kirchhoff's Voltage Law between the two plugs and computes real and reactive power for all three phases.

3.2 Basic one-phase components

The basic one-phase RLC components are assorted in the package Complex.SinglePhase.Basics. The inductor's model (see **Fig. 4**) reads e.g. without any annotations and comments:

```
model Inductor
  extends
  Complex.SinglePhase.Interfaces.OnePort;
  parameter Real L=1;
equation
  vRe = -omega*L*iIm;
  vIm =  omega*L*iRe;
end Inductor;
```

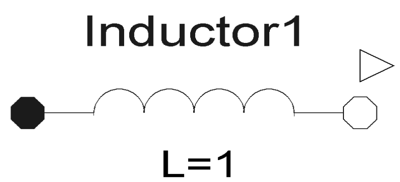

Figure 4: Icon for comlex inductor

If the inductance is not constant but depends on time or some other physical quantity like a mechanical coordinate then the model VariableInductor has to be used. In this model, the inductance is governed via an input signal.

```
model VariableInductor
  extends
    Complex.SinglePhase.Interfaces.OnePort;
```

```
  Modelica.Blocks.Interfaces.RealInput L;
equation
  vRe = -omega*L*iIm;
  vIm =  omega*L*iRe;
end VariableInductor;
```

A complete list of the `Basics`-package reads:

- Ground
- Resistor
- Inductor
- Capacitor
- Conductor
- Transformer
- IdealTransformer
- VariableResistor
- VariableInductor
- VariableCapacitor
- VariableConductor
- VariableTransformer

Some one-phase source components can be found in the package `Complex.SinglePhase.Sources`. Presently, voltage sources and current generators are implemented with

- constant amplitude and phase,
- amplitude and phase governed by input signal.

Additionally, power sources with given power (constant or governed by input signal) are available. Furthermore, the package `Complex.SinglePhase` contains some sensors for "measuring" the phasors (rms value, phase) of voltage or current. Finally, the power quality sensor (`PQ_sensor`) can be used to determine all interesting values concerning the power and its quality.

3.3 Basic multi-phase components

The basic multi-phase RLC components are assorted in the package `Complex.MultiPhase.Basics`. These definitions can be used in symmetric multi-phase grids of arbitrary number of phases. The package contains models for resistors, inductors, capacitors, and conductors each with constant constitutive parameters or with parameters governed by input signals. For comparability, the inductor's model with constant inductance is given here:

```
model Inductor
  extends
    Complex.MultiPhase.Interfaces.TwoPlug;
  parameter Real L[m]=fill(1,m);
  Complex.SinglePhase.Basics.Inductor
    inductor[m](final L=L);
equation
  connect(inductor.p, plug_p.complexpin);
  connect(inductor.n, plug_n.complexpin);
end Inductor;
```

Moreover, elements for realizing star connections or delta connections as well as for connecting multi-phase and single-phase components together are included in the package. Finally, some sources and sensors are available, too.

3.4 Electric induction machine

In the presented library, a model of an electric induction machine has been implemented, too. To this end, the package `Complex.Machines` was created. An induction machine can be regarded as consisting of a mechanical and an electrical subsystem. Specific interactions take effect between the two subsystems. The model of the induction machine combines a transient (time-domain) part of the mechanical subsystem with a phasor-domain part of the electrical subsystem. The model is only valid for analysing the machine within steady-state or quasi-stationary mode of the electrical subsystem. **Fig. 5** shows the well-known steady-state equivalent circuit for one phase of an induction machine. In this diagram, $\underline{V}_N$ is the voltage

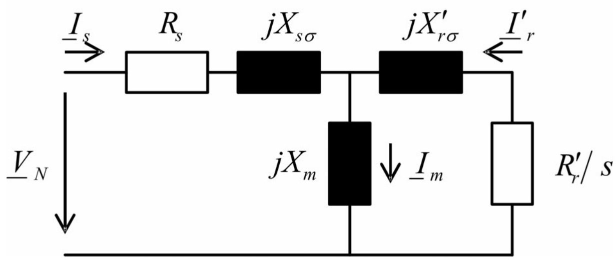

Figure 5: Induction machine's equivalent circuit

phasor of the net, $\underline{I}$ denotes a current phasor (subindex s stands for the stator, subindex r stands for the rotor, m denotes the main reactance, σ refers to the stray reactances). R and $X = \omega L$ are ohmic resistance and inductive reactance, respectively. The slip between electrical angular frequency ω and rotor's angular velocity ω_m is denoted by s, while $s = (\omega - \omega_m)/\omega$. (Please note that this model is a time phasor description not using any space phasors.) Furthermore, it shall hold $X_s = X_m + X_{s\sigma}$ and $X_r = X_m + X_{r\sigma}$. Finally, the stray coefficient can be determined from the reactances according to $\sigma = 1 - \sqrt{X_m^2/(X_s X_r)}$. Using some simplifying assumptions (e.g. R_s sufficient small) and denoting the number of phases by n, the torque produced electrically reads (see e.g. [13])

$$T = n\frac{V_N^2}{\omega}\frac{(X_h/X_s)^2}{(R_r'/s)+(s/R_r')(\sigma X_r)^2}. \qquad (22)$$

Slip s and torque T are the signals which connect the mechanical and the electrical subsystem. Considering the definitions of coupling signals in chapter 2.4 (see **Fig. 3**), the slip is the I-signal and the torque is the O-signal. Hence, the slip must not vary too fast, because one must ensure at each point in time that the electrical subsystem works within the quasi-stationary mode. (An automatic monitoring of this demand is not implemented in the library at the moment but would be a nice feature.) Under this assumption, the torque is always calculated correctly.

4 Examples

4.1 Electric circuit with varying resistance

Fig. 6 shows a simple example circuit containing a `VariableResistor`-component. The voltage source V works with constant amplitude and zero phase. The resistance R3 is a ramp function of time ($10^{-6}\ldots 5\Omega$). **Fig. 7** shows the rms values of the voltage drops across resistor R2 and the inductor as well as the source voltage. The voltages across R2 and L

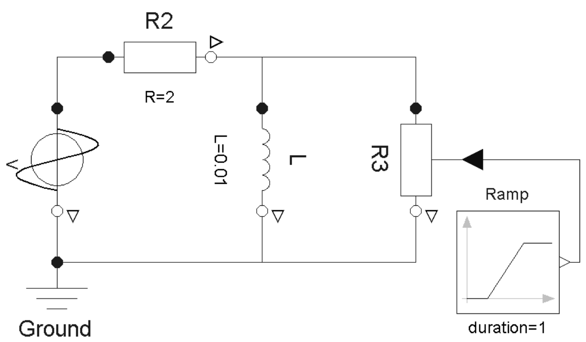

Figure 6: Example circuit

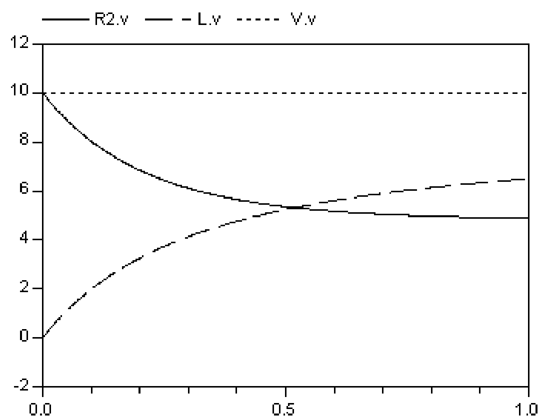

Figure 7: Voltages drops across R2 and L, source voltage

depends on time. But this curve is not a transient response. It is rather a sequence of quasi-stationary states spread over time axis. To prove this statement, one shall have a look at the time constant of the curves. This constant is about 0.5 s which is more than $10T$ because a nominal frequency of 50 Hz is used. The real and imaginary part of the voltage drop across the inductor is plotted in **Fig. 8**. This diagram shows the variation of phase within the sequence of quasi-stationary states.

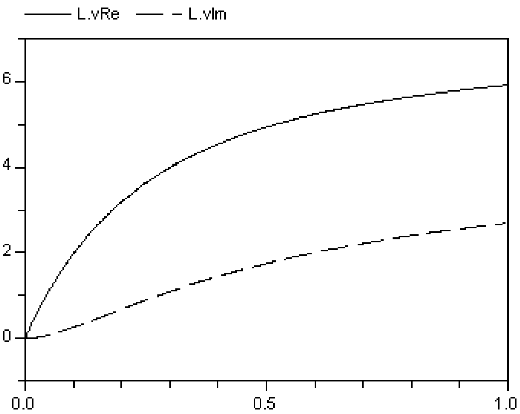

Figure 8: Real and imaginary part of voltage across L

4.2 Induction machine in quasi-stationary mode

This example deals with a three-phase induction machine. The test setup is shown in the schematic diagram of **Fig. 9**. The machine's electrical subsystem

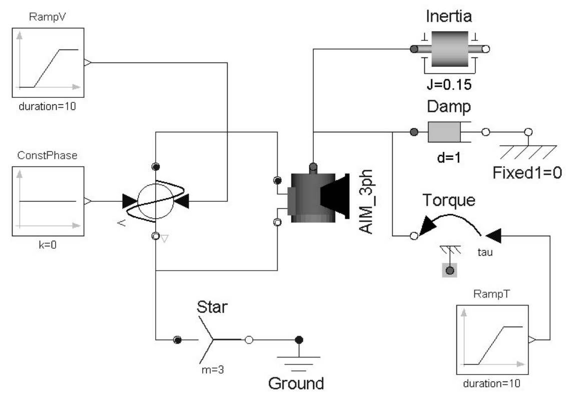

Figure 9: Schematic diagram

is connected to a "three-phase voltage source" which is the same as being connected to three single source components. The three-phase source works with variable amplitude and phase each governed by one input signal. During the simulation, the amplitude increases along a ramp function (see **Fig. 10**, solid line) while the phase remains at zero. On the mechanical side, the shaft of the induction machine is connected to an additional inertia, a damper component, and an applied torque. Inertia and damping constant have fixed values. The torque is governed by a ramp function which starts at a simulation time of 20 s (see **Fig. 10**, dashed line).

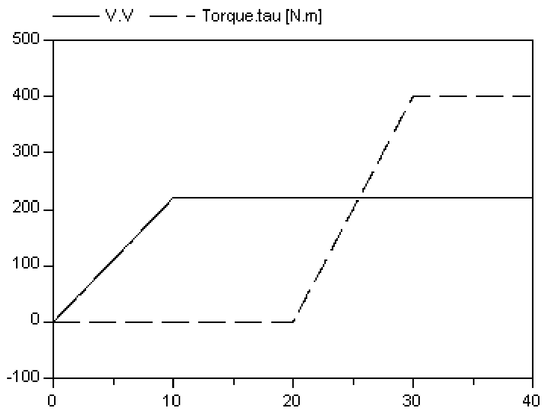

Figure 10: Inputs: source voltage and applied torque

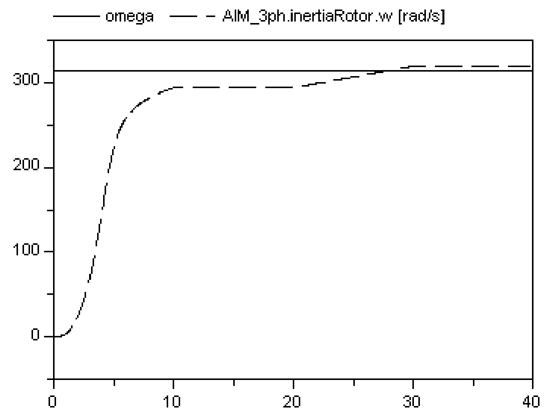

Figure 11: Electrical and mechanical frequencies

In the first time interval (between 0 and 20 s), the induction machine works as a motor. The increasing stator voltage causes an increasing acceleration of the shaft. Because of the mechanical damping, the angular velocity – i.e. the shaft velocity – (see **Fig. 11**, dashed line) finally reaches a value which is less smaller than the electrical angular frequency (see **Fig. 11**, solid line). Hence, the slip has only positive values in the first time interval. **Fig. 11** additionally shows that the simulation produces a transient response of the shaft's angular velocity. The time constant during the highest acceleration is about 0.2 s which equals $10T$. Therefore, the electrical subsystem is still in the quasi-stationary mode.

In the second time interval (between 20 and 40 s), the shaft is accelerated by an increasing torque which is additionally applied to it. At one moment in time, the shaft's angular velocity becomes higher than the electrical angular frequency. Beginning with this time instant (approximately 28 s), the induction machine is working in generator mode. Hence, the slip goes below zero. The transition of the working modes is characterized by a change of the sign of the difference between both frequencies or – which is the same – by a change of slip's sign. The transient process in the second time intervall is much slower than in the first interval. Therefore, the electrical subsystem is in the steady state or in the quasi-stationary mode during the complete simulation.

Finally, **Fig. 12** shows the electrical active power of one phase of the voltage source component and the mechanical power of the complete induction machine.

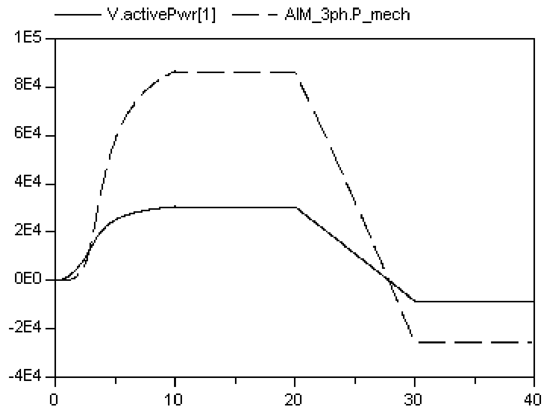

Figure 12: Electrical active power

Both curves have analogous shapes. After the transient response is faded away, the mechanical power is three times higher than the electrical power (because a three-phase machine is under consideration). In the first part of the simulation, the power curves have positive values which means that the source component produces energy while the motor consumes it. After a simulation time of about 28 s, the curves go below zero. This fact indicates that now the induction machine produces energy which is fed back into the electric net via the source component.

4.3 Example for domain switching

The switching between time domain and phasor domain and vice versa will be explained on a simple circuit containing a sinusoidal voltage source, an ohmic resistor, and a capacitor. **Fig. 13** shows the time-domain model (on the left hand side) and the phasor-domain model (on the right hand side). The left circuit remains in time domain during the complete simulation. The right circuit is modelled in time domain at the beginning. At an arbitrary point in time ($t_1 = 0.177$), the modelling domain of the right subsystem is changed to phasor domain (T-P changeover). Now, the phasor-domain description is used until $t_2 = 0.377$ (again chosen arbitraryly). At this time instant, the modelling domain is changed back to time domain (P-T changeover). From t_2 to the end of simulation, the time-domain model is used. The component's parameters are identical in both circuits ($R = 10\Omega$, $C = 1\text{mF}$). Both sources are feeding a voltage of 1 V at $t = 0$ increasing at a rate of 0.5V/s. Because of the low increasing rate, the circuits are always in a quasi-stationary mode.

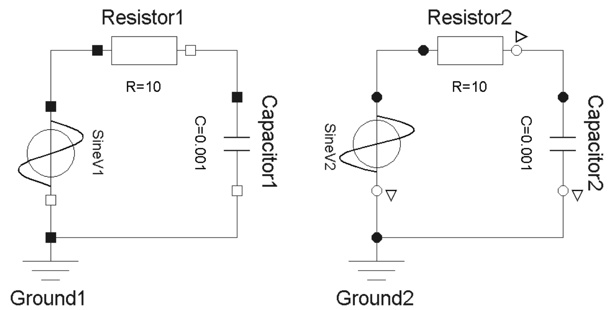

Figure 13: Schematic of domain-switching example; left: always within time domain, right: switching from time domain to phasor domain (T-P) and reverse (P-T)

Some simulation results are shown in the following figures. The first three diagrams contain the voltage drop Vcref across the capacitor of the permanent time-domain system and, from the switching system, the sinusoidal voltage drop (first and third interval) or the amplitude of the voltage phasor (second interval), respectively (both denoted by Vc). **Fig. 14** shows the complete simulation progress, while **Fig. 15** and

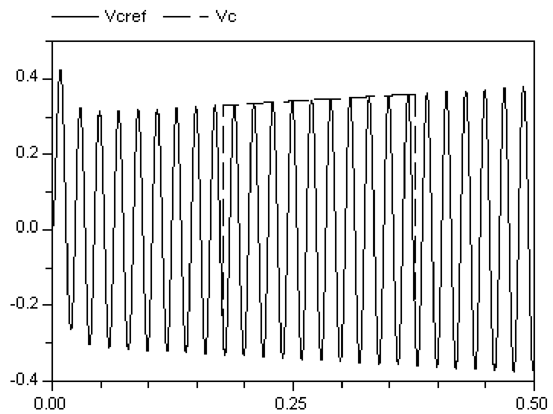

Figure 14: Voltages; Vcref: always in time domain, Vc: switching T-P and P-T (showing amplitude in P-domain)

Fig. 16 represent the details around the switching instants. Using the time-domain model for the switching circuit, both curves (Vcref and Vc) are identical. If this circuit is described within the phasor-domain then the dashed curve is nearly constant showing only the amplitude of Vc.

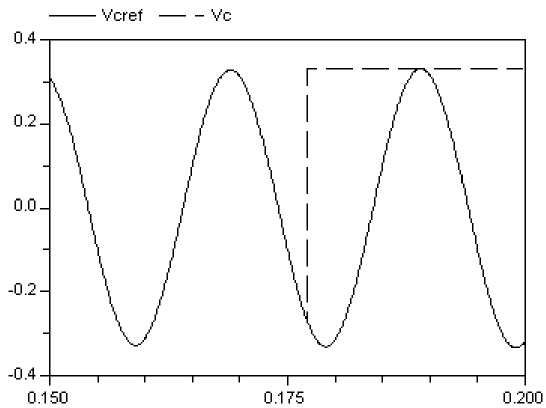

Figure 15: Zoomed voltages; T-P changeover

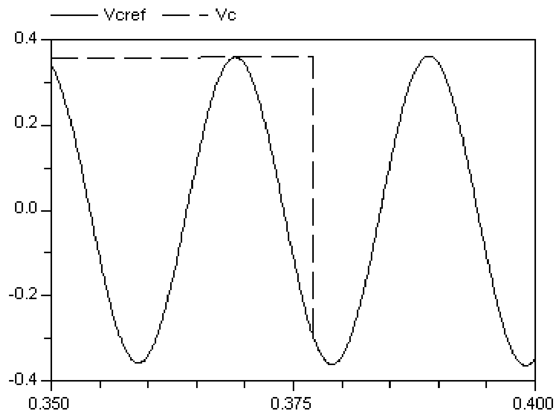

Figure 16: Zoomed voltages; P-T changeover

The last three diagrams show the current flowing through the permanent time-domain circuit (Iref) and, from the switching system, the sinusoidal current (first and third interval) or the amplitude of the current phasor (second interval), respectively (both denoted by I). **Fig. 17** shows the complete simulation progress, while **Fig. 18** and **Fig. 19** represent the details around

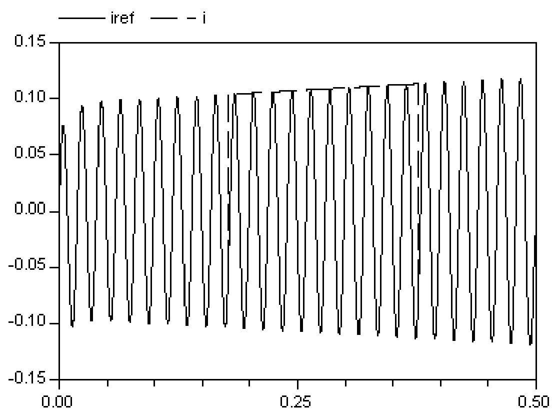

Figure 17: Currents; Iref: always in time domain, I: switching T-P and P-T (showing amplitude in P-domain)

the switching instants. In these diagrams, both currents are identical, too, if the time-domain model is applied to the switching circuit. If the phasor-domain model is valid then the dashed curve is nearly constant showing only the amplitude of I.

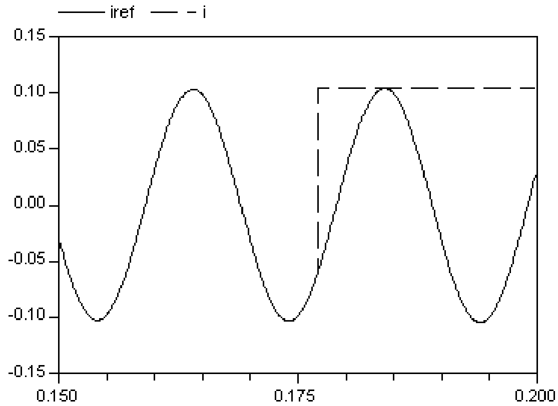

Figure 18: Zoomed currents; T-P changeover

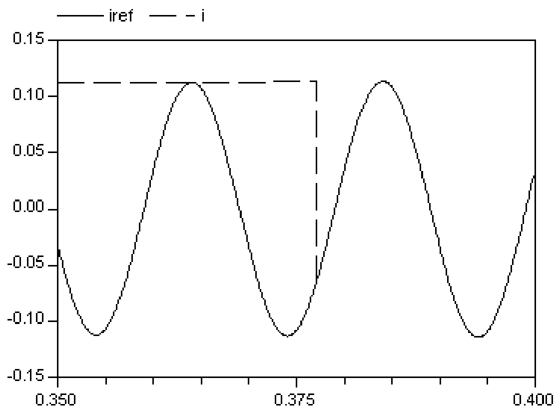

Figure 19: Zoomed currents; P-T changeover

5 Summary

This paper deals with the quasi-stationary AC analysis using phasor-domain models within Modelica. First, the term "quasi-stationary mode" is declared. Hereafter, some fundamentals concerning phasor description are repeated. Using such phasors, a possible implementation of AC circuits within Modelica is proposed. The main content of a corresponding new Modelica-library is introduced. This library can be used for modelling AC circuits in a very efficient way. Furthermore, some investigations of coupling time-domain models and phasor-domain models are presented in the paper. Hence, the introduced Modelica-library can also be used for modelling electromechanical drives with electrical AC subsystems under some assumptions. This fact is examplified by modelling an induction machine.

Studying AC systems for long time periods within the time domain is hardly possible. Therefore, it may be of interest to switch between a time-domain model and a phasor-domain model and vice versa in an appropriate manner. This scenario is shortly concerned in the paper.

Finally, simulation results of three examples are given proving the principal capabilities of the library.

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