Acausal Modelling of Helicopter Dynamics for Automatic Flight Control Applications

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Abstract

In the preliminary design stages of helicopter autopilots computationally affordable and mathematically simple dynamic models are needed to perform approximated performance assessments. In this paper, the structure of a modular, acausal and reconfigurable helicopter simulator is described, showing how the innovative characteristics of Modelica language can be employed in order to simplify the implementation of the single components and to allow the optimization of the simulator architecture. The overall model makes use of the existing Modelica MultiBody and Mechanics.Rotational libraries.

Keywords: helicopter; flight mechanics; simulation; AFCS

1 Introduction

The mathematical modelling of rotorcraft dynamics is a very difficult task that has represented a challenge for many researchers since age '60s. The availability of performant computers and a deeper theoretical knowledge in the late 80's dramatically improved the results achieved in this field. Nowadays, this research area is extremely wide, as it entails advanced studies in computational fluid dynamics (CFD) and flexible multibody dynamics as well. However, Heffley et al. [11] and successively Padfield [3] underlined how, for handling analysis and for the design of common mid-low bandwidth automatic flight control systems (AFCS), the needed mathematical model of the plant shouldn't be too much complex (at least in the early stages of the project), because the experimental validation turns out to be much simpler with a reduced number of uncertain parameters and because the fundamental aeromechanical phenomena can be reasonably matched with simplified, first principle based, models. Furthermore, this kind of model is generally easier to use (requiring a relatively small set of parameters), computationally less expensive and then it's more suitable for real-time applications. This is the reason why a consistent number of researchers have devoted themselves in the last twenty years to the development of so-called minimum complexity helicopter math models (see [8], [11] for more details), which are able to correctly predict the prevailing phenomena involved in helicopter handling and control. The most recent result, in that sense, is given in Padfield's book [3], where the definition of Level 1 helicopter dynamic model is given.
In this paper, a Modelica implementation of a Level 1 helicopter dynamics model is presented, showing how the peculiar characteristics of Modelica language may be profitably used in order to make the implementation as natural as possible. The paper is organized as follows: in section 2 a brief overview of main rotor modelling techniques is given; in section 3 the Modelica implementation of the proposed helicopter model is described, followed in section 4 by simulation studies; at the end of the paper, concluding remarks and future developments are outlined.

2 Overview of basic main rotor dynamic modelling

In this paragraph, a synthetic description of a main rotor dynamic model suitable for flight mechanics simulation is provided, according to Level 1 model definition. The resulting mathematical model may be used not only for control synthesis purposes, but also for preliminary performance calculations. Main rotor model is without any doubt the most important and complex helicopter subsystem, being a fundamental

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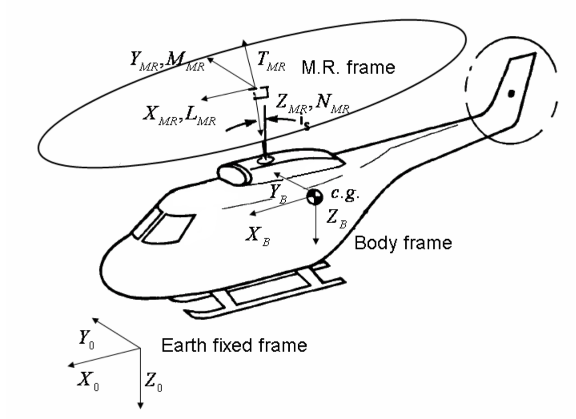

Figure 1: Helicopter reference frames [8]

source of lift and controllability for the aircraft, transferring prevailingly aerodynamic forces and moments from the rotating parts (blades) to the non-rotating frame (that is the fuselage); pilot and AFCS have direct control of main rotor thrust amplitude through collective blade pitch and indirect control of thrust direction by means of cyclic blade pitch (because the flapping rotating blades act together as a gyroscope). The pitch of each blade is varied with particular mechanical devices (typically swashplate or "spider" assemblies, [1]). Since control-oriented helicopter simulators usually describe main rotor making use of analytically integrated loads, in order to provide deep understanding of the underlying physics and a significant simplicity, it's evident that a so-called Level 1 main rotor model is more suitable for a general, modular and reconfigurable simulator than a complex CFD-based "numerical" model, where the aerodynamic loads are obtained integrating the pressure distribution upon each blade and, in the same way, the inertial loads are computed using complex codes for flexible multibody systems [14]. This last approach becomes necessary when high bandwidth control actions (for example HHC - high-harmonic-control - techniques for the active control of vibrations [5]) or detailed analyses must be performed, as underlined in [15]. In the following, we shall provide a simplified description of the main rotor mathematical model used in our simulator: the interested reader can find much more details in the specialized literature [13], [17]. Let's assume the following hypotheses:

- Compressibility, stall and reverse flow effects are neglected.
- The possible different blade flap retention arrangements (teetering, articulated and hingeless rotor) are described with the unifying centre-spring equivalent rotor theory.
- Fast lead-lag dynamics is neglected.
- The blades are considered rigid (in case of hingeless rotors, flexibility is concentrated in the centre-spring).
- Aerodynamic loads are computed using simple blade element theory.

Introducing the MBC (Multi-Blade Coordinates) transformation, one can describe flapping dynamics (which is fundamental in helicopter handling and control) with reference to the non-rotating frame. Representing with β_i the flapping angle of i-eth blade, with N_b the blades number and with $\psi = \Omega t$ the rotor azimuth angle in wind-axes system (rotor angular velocity Ω is assumed constant), MBC transformation is defined as:

$$\begin{aligned}
\beta_0 &= \frac{1}{N_b}\sum_{i=1}^{N_b}\beta_i \qquad (1)\\
\beta_{0d} &= \frac{1}{N_b}\sum_{i=1}^{N_b}\beta_i(-1)^i\\
\beta_{jc} &= \frac{2}{N_b}\sum_{i=1}^{N_b}\beta_i\cos j\left[\psi+\frac{2\pi}{N_b}(i-1)\right]\\
\beta_{js} &= \frac{2}{N_b}\sum_{i=1}^{N_b}\beta_i\sin j\left[\psi+\frac{2\pi}{N_b}(i-1)\right]
\end{aligned}$$

Neglecting periodic terms, which influences only vibrations, and the differential coning β_{0d}, which is reactionless and, in any case, null for N_b odd, the whole rotor disc configuration can be described using only the so-called coning mode β_0 and the first two cyclic modes β_{1c}, β_{1s} (representing longitudinal and lateral disc tilt angles, respectively) according to the following expression:

$$\beta(\psi,t) = \beta_0(t) + \beta_{1c}(t)\cos(\psi) + \beta_{1s}(t)\sin(\psi) \qquad (2)$$

Using the vector representation $\bar{\beta} = \{\beta_0, \beta_{1c}, \beta_{1s}\}$, the flapping equations for the dynamics of a generic N_b-bladed rotor[1] can be expressed in the form [3],[12]:

[1] For a two bladed teetering rotor, β_0 =const must be assumed

$$\ddot{\overline{\beta}} + C_f\dot{\overline{\beta}} + D_f\overline{\beta} = H_f \quad (3)$$

The matrix C_f, D_f and the vector H_f are complex function of rotor system parameters (in particular, blade Lock number γ, representing the ratio between blade aerodynamic and inertial load, and equivalent spring stiffness K_β), flight conditions (mainly μ, advance ratio, representing the air velocity lying in rotor disc plane adimensionalized with respect to blade tip speed), blade pitch angle and aerodynamic inflow distribution. These last two contributions deserve more attention; in particular, blade pitch can be expressed in a way similar to (2)

$$\theta(\psi,t) = \theta_0(t) + \theta_{1c}(t)\cos(\psi) + \theta_{1s}(t)\sin(\psi) \quad (4)$$

where the three system inputs θ_0, θ_{1c} and θ_{1s} are, respectively, the collective blade pitch, the lateral cyclic pitch and the longitudinal cyclic pitch. The prediction of the aerodynamic inflow (that is the flowfield induced by the rotor at the rotor disc) is a really complex task, as it involves the dynamic description of a completely three-dimensional aerodynamic field. In flight mechanics applications, anyway, simple mathematical model are often used for this task, varying from the simple theoretical static uniform momentum theory to more complex dynamic wake models. In particular, all the dynamic models derived from the original work of Pitt and Peters [10] represent a good compromise between simplicity, physical consistency and correspondence with experimental flight data and they are suitable for flight mechanics and control applications. These models typically describe dynamic inflow with a three states approximation, in order to correctly predict the first harmonic distribution of induced velocity on the rotor disc in maneuvered flight:

$$\lambda(r,\psi,t) = \lambda_0(t) + \lambda_{1c}(t)\frac{r}{R}\cos(\psi) + \lambda_{1s}(t)\frac{r}{R}\sin(\psi) \quad (5)$$

where R is the rotor radius and the pair (r,ψ) uniquely represents the position of a point on the rotor disc. The equations for the inflow dynamics are commonly expressed in adimensional form as:

$$[M]\left\{\begin{array}{c}\dot{\lambda}_0\\ \dot{\lambda}_{1s}\\ \dot{\lambda}_{1c}\end{array}\right\} + [L]^{-1}\left\{\begin{array}{c}\lambda_0\\ \lambda_{1s}\\ \lambda_{1c}\end{array}\right\} = \left\{\begin{array}{c}C_T\\ C_L\\ C_M\end{array}\right\} \quad (6)$$

The coefficients (C_T, C_L, C_M) represent the adimensionalized resultant aerodynamic thrust, rolling moment and pitching moment, respectively, in non-rotating frame. The expressions for the mass-apparent matrix M and the matrix L can be found, for example, in [9],[10]. In order to solve this complex mathematical problem, the expressions for the resultant forces and moments transmitted by the rotor to the fuselage need to be computed. Integrating analytically the aerodynamic and inertial loads acting on each blade and summing over all the blades, closed-form expressions for the rotor forces and moments X_w, Y_w, T, L_w, M_w, N can be derived; these formulas (here non reported for brevity, see [3],[8] for more detail) included vibratory terms, which are usually neglected in flight mechanics analyses, and quasi-steady terms, which are, on the contrary, of chief interest. The prefix w stands for "wind", in order to remember that these components need to be transformed from the hub-wind system (aligned with relative air velocity) to the shaft-axes system taking into account the sideslip angle β_w, before assembling the overall system dynamics. From this analysis it's clear that also the implementation of a basic complexity main rotor model in a traditional simulation environment may result a very difficult and time-consuming task, involving the solution of a fully coupled mathematical problem. In the next section, a Modelica implementation of a control-oriented helicopter dynamic model is described, showing the remarkable simplification allowed by the modelling paradigms offered by Modelica language.

3 A Modelica comprehensive control oriented helicopter simulator

In this section, a control-oriented helicopter dynamic model developed with Modelica language is presented. This model has been implemented employing the well-known analytical results published in [3],[8],[11] which constituted the starting point for a significant number of helicopter flight simulators, in military and research fields. The overall helicopter model takes advantage of the unique object-oriented features of Modelica language and it includes the following submodels:

- Atmosphere model
- Main rotor dynamics
- Tail rotor dynamics

- Airframe rigid mechanics
- Fuselage and empennages aerodynamics
- Turboshaft engine simplified dynamics

The Atmosphere model is declared with **outer** keyword, in order to make it accessible from all the helicopter components; it implements a U.S. Standard Atmosphere model for the variation of air pressure, density and temperature with altitude, and it allows to define a uniform wind disturbance. The model can be extended in order to include also a stochastic turbulence model (for example based on Dryden's turbulence spectrum [4]). Since all the helicopter aerodynamic components need the updated value of the density ρ and since the small variations of density from a point to another of the aircraft do not justify a "local" evaluation of this quantity with a function, we have decided to compute it once for the whole helicopter, making reference to the altitude of its center of mass. A significant advantage found using Modelica for this application regarded the computation of local airspeed at different points of the helicopter; as pointed out by Looye and Moorman [2] speaking about their FlightDynamics library, the local airspeed is given not only by the inertial velocity of the center of mass and by the wind components, but it's also influenced by aircraft angular velocities. Local airspeed at a generic point p of the helicopter can be expressed as:

$$\vec{V}_a(p) = \vec{V}(p) - \vec{V}_w(p) - \vec{V}_{dw}(p) \tag{7}$$

where $\vec{V}_a$ is the local airspeed, $\vec{V}$ is the local inertial velocity, $\vec{V}_w$ is the local windspeed (in our case it's assumed to be uniform) and $\vec{V}_{dw}$ represents the velocity of the airflow determined by downwash effects. Employing the **FixedTranslation** components of the MultiBody library [7], this problem is easily solved, since $\vec{V}$, derived from the MultiBody frame relative to the considered aerodynamic component, takes automatically into account the velocity transport effect due to rigid rotations. On the other side, this procedure avoids user to manually specify (possibly introducing implementation bugs) the transport laws of forces and moments from aerodynamic components (for example from the center of pressure of the different lifting surfaces or from the hub reference system of main and tail rotor) to the center of mass reference system, as commonly happens in flight simulators implemented with low-level languages (such as Fortran or C). Eventual downwash effects (as those due to main and tail rotor inflow) have been included introducing a dedicated **inflow connector**, which is declared as following:

```
connector Inflow_in
"External velocity field with harmonic
distribution"
input Modelica.SIunits.Velocity v0
"Average inflow component";
input Modelica.SIunits.Velocity vs
"Sinusoidal inflow component";
input Modelica.SIunits.Velocity vc
"Cosinusoidal inflow component";
end Inflow_in;
```

The same holds for the dual connector `inflow_out`, but the inflow components are declared as `output`. The causal nature if this connector is obviously made necessary by the simple mathematical modelling of downwash effects carried out in flight mechanics.
Thanks to the innovative features of Modelica language, rotor dynamics can be implemented in the most natural way, declaring the equations as found in technical reports and specialized books (as those summarized in the preceding section) without requiring any error-prone and time consuming by hand manipulation of the analytical expressions. For example, let's consider the tail rotor dynamics: tail rotor is simpler to describe than main rotor, because it has no cyclic inputs but only a collective pitch input; moreover, its high rotating speed makes the flapping and inflow dynamics so fast that they are usually neglected. Static inflow theory is therefore suitable for the computation of tail rotor induced velocity and thrust:

$$\begin{aligned} \lambda_i &= \frac{V_i}{\Omega_{tr} R_{tr}} = \frac{C_T}{2\sqrt{\mu^2 + \left(-\frac{v}{\Omega_{tr} R_{tr}} - \lambda_i\right)^2}} \\ C_T &= C_T\left(\lambda_i, \mu, \theta_{tr}\right) \end{aligned} \tag{8}$$

where $V_i, C_T, \Omega_{tr}, R_{tr}, \mu, v$ represent, respectively, tail rotor induced velocity, thrust coefficient (adimensionalized thrust), rotating speed, radius, advance ratio and transversal airspeed (directed as the helicopter y-axis); furthermore, θ_{tr} symbolizes the tail rotor pitch input. The expression (8) is an implicit equation, because the inflow depends on the thrust and the thrust depends on the inflow; using traditional, causal, simulation tools or low level languages, the user should solve this equation implementing by hand the code for a Newton's iterative scheme or (as suggested in [11]) introducing

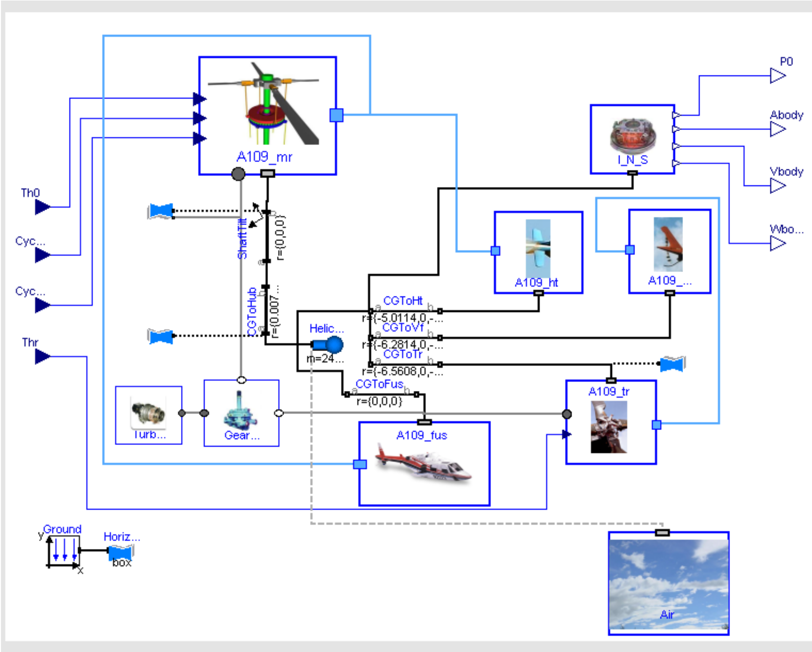

Figure 2: Helicopter simulator layout

unphysical algebraic loop by means of common I/O blocks. With Modelica language, this is no more necessary, since Dymola provides analytical manipulation of these expressions, and the user can introduce the expressions like (8) (much common in aerodynamic context) without any modification.

Aerodynamic forces and moments generated by empennages and fuselage are computed employing look-up-tables (Modelica **CombiTable** blocks) and taking into account downwash effects due to main and tail rotor with inflow connectors. In the case of tail rotor, uniform inflow implies that $v_0 = V_i$ and $v_s = v_c = 0$. An important effect to predict, both for trim and dynamic analysis, happens when the impingement of main rotor wake on horizontal tail is realized. For this reason, horizontal tail model checks continuously the value of wake skew angle, which is defined as:

$$\chi = \tan^{-1}\left(\frac{\mu}{\lambda_0 - \frac{w}{\Omega R}}\right) \tag{9}$$

The aerodynamic coefficients used in fuselage model have been derived from the technical document [16], which reports wind tunnel data for different fuselage shapes.

Helicopter rigid mechanics is implemented using a standard **Body** component of MultiBody library and specifying aircraft inertial properties. The rigid body is connected to the different components of the airframe with rigid translations, as said before.

Helicopter main and tail rotors are designed for providing optimal performances at a constant, nominal, blades angular velocity; for this reason, engine automatic regulation is used in order to guarantee the right value of RPM, no matter how required power is. The **Mechanics.Rotational** library has been profitably used for the implementation of a simplified turboshaft engine dynamics model, connected to the main rotor block and to the tail rotor system through an ideal gearbox model. The torque absorbed at engine shaft is given by:

$$Q = Q_{mr} + I_{mr}\left(\dot{\Omega} - \dot{r}\right) + k_T Q_{tr} + Q_{acc} \tag{10}$$

where Q_{mr} is the torque due to rotor aerodynamic in-plane components (computed in main rotor block), I_{mr} is the rotor moment of inertia about shaft axis, r is the helicopter yaw rate, k_T the gear-box reduction ratio, Q_{tr} the torque required at tail rotor shaft and, finally, Q_{acc} represents the torque due to accessories. Observe that the torque balance (10) is realized in Modelica in a very physical way connecting rotational flanges. Engine dynamic response from fuel flow to shaft power has been modelled as a simple first order lag, according to [8], while the RPM governor has been implemented with an analog PID block. Figure 2 shows the Dymola graphical layout of the helicopter simulator. On the left the four helicopter inputs (m.r. collective pitch, m.r. longitudinal cyclic pitch, m.r. lateral cyclic pitch and t.r. collective pitch) are visible, while on the right the output signals correspond to measured Euler/Cardan angles, pitch rates and airspeeds. Light thick connections represent aerodynamic interferences (downwash effects). Eventual primary mixer (for the decoupling of pitch and roll control channels) and interlink subsystems can be easily added to this basic simulator. Furthermore, specialized graphical shapes have been employed for the 3D visualization of the helicopter trajectories (Figure 3).

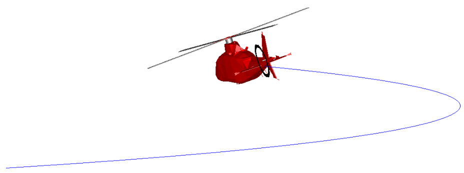

Figure 3: A109 helicopter performing a steady coordinated turn

4 Simulation study

As a case study, the technical data of an *Agusta A109mkII* helicopter have been considered [11]. The complete helicopter system is fully specified in our simulator inserting only 69 parameters (excepting the coefficients of look-up-tables). In the following, the results deriving from two different simulation studies are reported.

4.1 Trim analysis

In this test, the helicopter model has been trimmed for different values of airspeed in both longitudinal and lateral flight conditions. Figures 4,5 report plots showing trim data values for angular attitudes, command inputs and required power for different trim conditions.

These results have been compared with the real flight data of the same helicopter model [6],[11], observing generally a very good agreement. Only a little overestimation of tail rotor authority has been observed for high speed flight, due probably to complex aerodynamic interactions between main rotor wake and tail rotor flowfield.

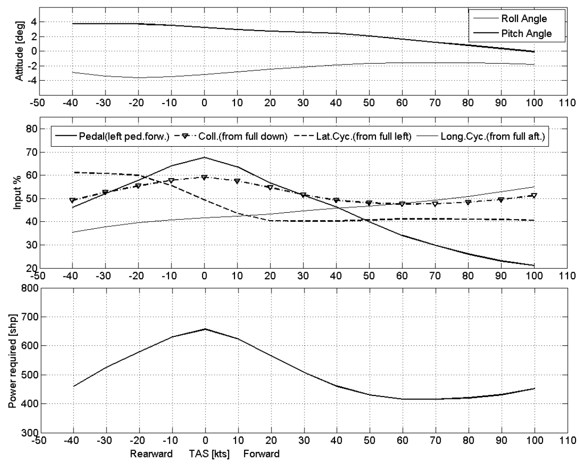

Figure 4: Longitudinal flight - trim data

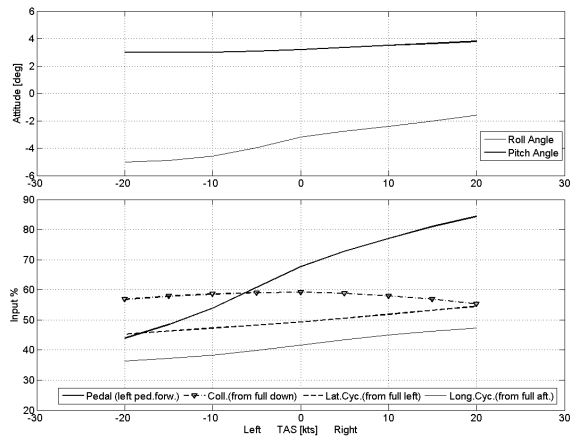

Figure 5: Lateral flight - trim data

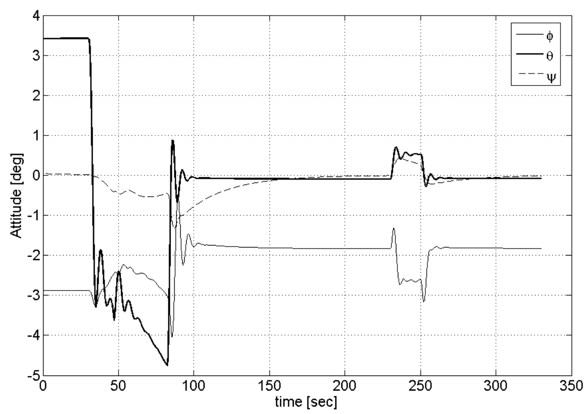

Figure 6: Helicopter attitudes

4.2 Piloted flight

Introducing a very simple AFCS control structure [3], we could easily reproduce the presence of a standard autostabilisation equipment plus a pilot (or an autopilot alone), in order to perform realistic transients. As an illustrative example, a simulation of a flight transient is reported: the helicopter starts hovering, then, for $t = 30$ sec the helicopter begins a longitudinal flight (sideslip $\beta_w = 0$) accelerating up to 100 knots (51.4 m/s, Figure 7). At $t = 230$ sec the helicopter starts a climb and it increases its altitude of 60m in 20 sec. The time history of three attitude angles is reported in Figure 6; the velocity increase from hover to 100 knots is achieved putting the helicopter nose down (the angle θ decreases) by means of a forward longitudinal stick displacement, as shown in Figure 8, where the longitudinal cyclic pitch reaches its steady-state trim value for 100 kts.

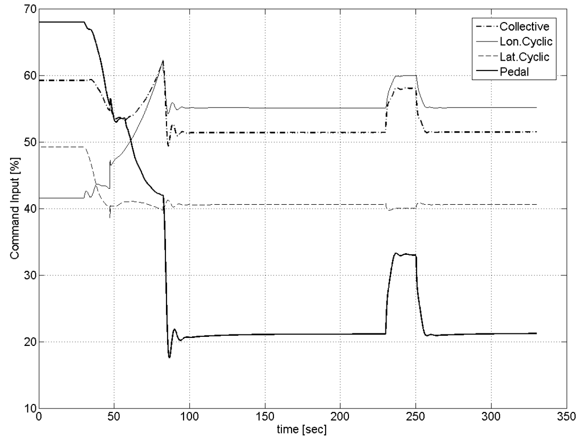

Figure 8: Helicopter inputs

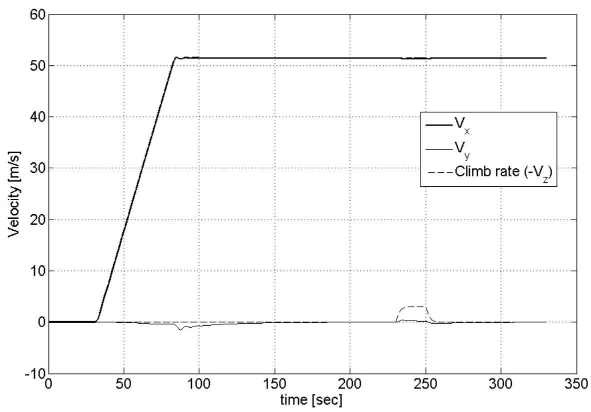

Figure 7: Helicopter velocities in earth-fixed frame

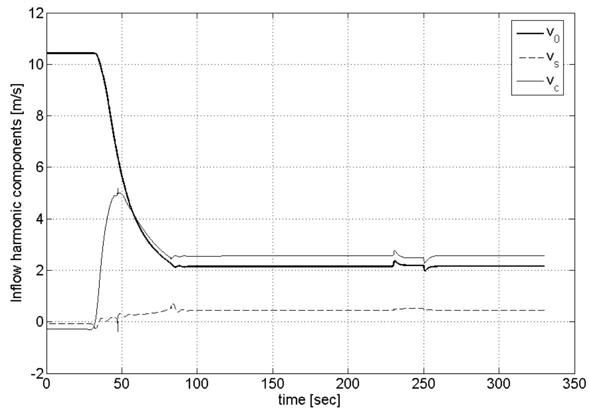

Figure 9: Main rotor inflow - harmonic components

Correspondingly, the virtual pilot uses pedals (tail rotor pitch) in order to hold heading during the transient, compensating main rotor torque. At $t = 230$ sec the collective is pulled up in order to impress a positive climb rate of about 3 m/s to the helicopter, until the new altitude is reached. Figure 9 reports the time history of the dimensional inflow states: as speed increases, the main rotor wake passes from a configuration where the inflow is approximately uniform ($v_s = v_c = 0$) to a harmonic distribution consistent with high speed forward flight. Finally, Figure 10 shows the required engine power during the transient.

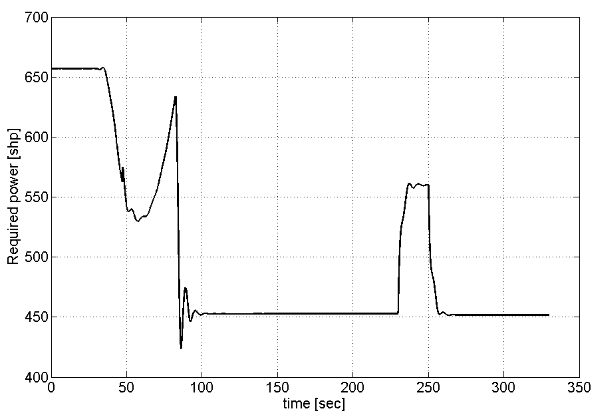

Figure 10: Required power

5 Conclusions and future work

In this paper, a simple parametric and reconfigurable helicopter dynamic model has been proposed, underlining the great advantages in terms of code readability and reusability deriving from the use of Modelica language paradigms and Dymola environment. As next work, the model validation will be performed. The validated model will be integrated with detailed electro-hydraulics actuator models (under development) and realistic redundant FCC (Flight Control Computer) architectures. The resulting simulator will be used for the assessment of performance degradation in case of actuators failures and for the support in analysis and design of new specific linear and nonlinear control algorithms.

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