

# Methods of Sensitivity Calculation Applied to a Multi-Axial Test Rig for Elastomer Bushings

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## Abstract

This paper presents methods of sensitivity calculation applied to a multi-axial test rig for elastomer bushings. In this context, the effect of parameter variations on system variables is analysed by using different tools. Aspects like modelling, modelling effort, computing time and accuracy are discussed.

This paper is based on results, which were developed in the Fraunhofer collaborative project “Computer Aided Robust Design (CAROD)”.

*Keywords: parameter sensitivity; Monte-Carlo simulation; elastomer test rig; DAE; Dymola; DASPK; MATLAB*

## 1 Introduction

Tolerances of material quality, manufacturing processes and assembly operations lead to scattering product properties. In mass production therefore more or less significant deviations of the desired “ideal” properties occur. Resulting from wear and degradation, also during product life cycle continuous changes of component and system characteristics take place.

The named uncertainties are only covered insufficiently in traditional development workflows of mechatronical products. In most cases, simulations without any parameter scatter are performed to optimise mechanical and mechatronical systems and to analyse their durability and reliability.

The objective of Robust Design is to analyse the effects of scattering component behaviour in early de-

velopment phases and to optimize products. This presentation will focus on sensitivity analysis, which typically is an initial task in robust design studies. The prior objective is to determine parameters, which highly affect the product behaviour.

The paper starts with a short description of the chosen technical example: A 3 DOF test rig for elastomer bushings. Subsequently, two multi-body-simulation models of the test rig using MATLAB and Modelica are presented, including a cross-comparison of the particular simulation results. In the following, the method of “sensitivity calculation” is introduced, which has conceivable potentials to lower the effort for sensitivity analyses. To validate the results, the implementation of a Monte-Carlo (MC) analysis is treated, which is based on repeated calls of the Dymola simulator. The results of this MC analysis are again compared with an MC analysis performed with MATLAB. Finally, based on the analysis results, the application and performance of the methods for sensitivity evaluation are discussed.

## 2 Technical Example: Test Rig for Elastomer Bushings

### 2.1 Application and Technical Description

The methods described in this paper are presented by example of a multi-axial test rig (figures 1 and 2), which is owned by Fraunhofer LBF. The test rig is mainly used for sign-off tests of automotive elastomer bushings considering service loads. Further on, the bushing’s dynamic transmission behaviour can be characterised.

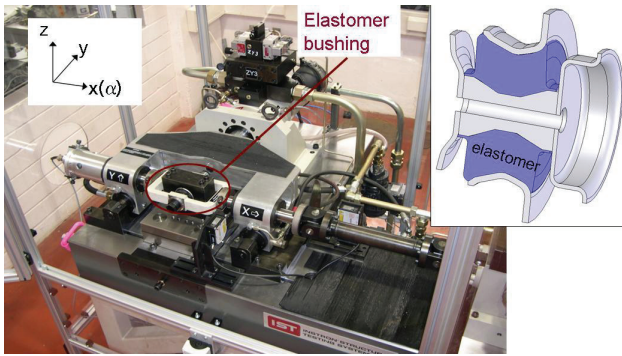


Fig. 1: Multi-axial test rig for characterisation and testing of elastomer bushings at Fraunhofer LBF and CAD model of a typical elastomer bushing (right)

Using servo-hydraulic actuation, variable cyclic loading of more than 100 Hz is feasible. Based on a transmission design with pre-stressed cardan joints, the load directions “axial ( $x$ )”, “lateral ( $y$ )” and “tor-sion ( $\alpha$ )” at the bolt can be realised isolated or in combination, respectively. To minimise wear and friction, hydrostatic linear guides between bolt and bail as well as bail and base plate are used. The movement of the bolt – and therefore the displacement of the elastomer bushing – is measured by sensors, which are positioned near the bushing’s clamping device. The reaction forces are measured by a piezoelectric measurement platform.

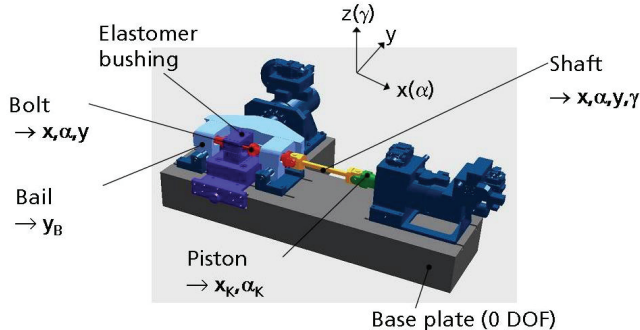


Fig.2: CAD model of the test rig, declaration of bodies and degrees of freedom (“absolute”)

## 2.2 Multi-Body-Simulation Model of the Test Rig and Simulation Tools

To analyse the dynamic behaviour of test rig mechanics, multi-body-simulations (MBS) are used (cp. [1]). In this context, the models typically include rigid bodies, which are linked by joints and force elements.

Corresponding, the MBS model of the elastomer test rig includes the rigid bodies “bolt”, “bail”, “piston” and “shaft”, while following joints and force elements are applied between the bodies (cp. figures 2 and 3):

- Bolt-bail: cylindrical joint  $x/\alpha$ , damping  $x/\alpha$  (hydrostatic bearings)
- Bail-base plate: prismatic joint  $y$ , damping  $y$  (hydrostatic bearing and friction of hydraulic cylinder)
- Bolt-base plate: 6 DOF, stiffness  $x/\alpha/y$  and damping  $x/\alpha/y$  (elastomer bushing)
- Bolt-shaft: cardan joint, no force element
- Shaft-piston: cardan joint, no force element
- Piston-base plate: cylindrical joint  $x/\alpha$ , damping  $x/\alpha$  (friction of hydraulic cylinder)

The transmission characteristic of the elastomer bushing is modelled by using the approaches

$$F_{E,x} = c_{E_{x3}} \cdot x^3 + c_{E_{x1}} \cdot x + d_{E_{x1}} \cdot dx/dt,$$

$$F_{E,y} = c_{E_{y3}} \cdot y^3 + c_{E_{y1}} \cdot y + d_{E_{y1}} \cdot dy/dt \text{ and}$$

$$M_{E,\alpha} = c_{E_{\alpha 3}} \cdot \alpha^3 + c_{E_{\alpha 1}} \cdot \alpha + d_{E_{\alpha 1}} \cdot d\alpha/dt,$$

which describe nonlinear stiffness and linear damp-ing behaviour for each load component. The parameter settings are based on measurement data derived by tests with a commercial elastomer bushing.

Comparable to the physical test rig, the MBS model is actuated by axial forces  $F_x$ , torsion  $M_\alpha$  (piston) and lateral forces  $F_y$  (bail).

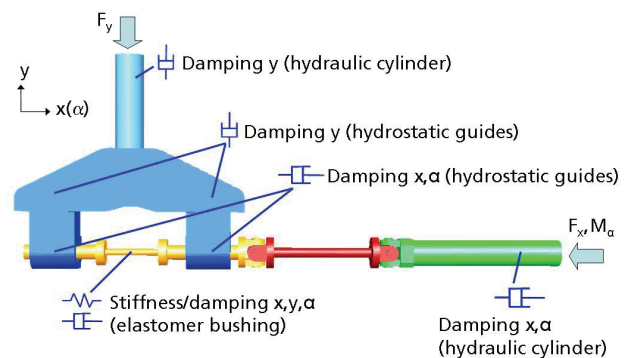


Fig.3: Multi-body simulation model of the test rig („top view“)

The MBS model was set up in MATLAB/SimMechanics as well as in Modelica (using a Dymola solver).

## 2.3 Comparison of Modelica and MATLAB Model

To compare the MBS models built in the MATLAB and Modelica environment, two test cases were defined. The first test case is characterised by sinusoidal forces and moments:

- $F_x = 3.0 \cdot 10^6 \cdot \sin(314 \cdot t)$  [N]
- $F_y = 3.4 \cdot 10^8 \cdot \sin(t)$  [N]
- $M_\alpha = 10^4 \cdot \sin(150 \cdot t)$  [Nm]

In contrast, the second test case includes the application of noise signals (figure 4). These signals were generated with a MATLAB script, which contains the rand command.

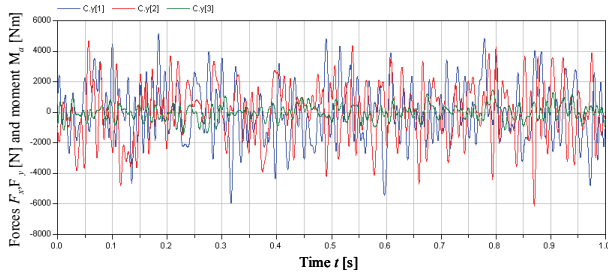


Fig. 4: Applied noise signals in test case 2

Figures 5 and 6 show the calculated displacements  $x$  and  $y$  [mm] as well as the torsion  $\alpha$  [rad] of the body “bolt” for the two test cases.

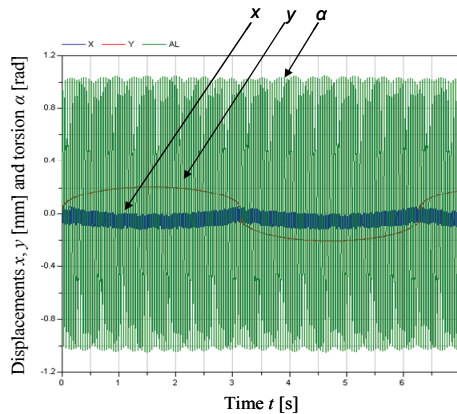


Fig. 5: Test case 1 - Solution (Dymola)

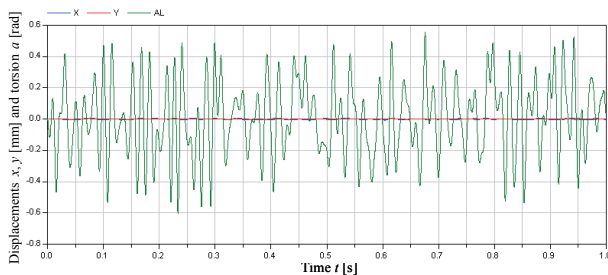


Fig. 6: Test case 2 – Solution (Dymola)

To compare the MBS models created in Modelica and MATLAB, the displacements  $x$  and  $y$  as well as the torsion  $\alpha$  of the body “bolt” were analysed in the time domain. Figure 7 shows the calculated displacements and torsion for test case 1, exemplarily. It is obvious, that both models lead to nearly identical results. This conclusion is affirmed by the results concerning case 2.

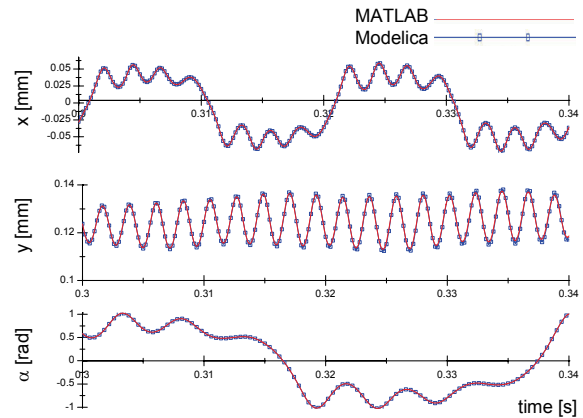


Fig. 7: Comparison of results from Modelica and MATLAB model (case 1)

## 2.4 Treated Scenario

The investigations target a virtual scenario, which describes the production of a small series of (only) theoretically identical test rigs. Due to manufacturing tolerances the test rig components will differ more or less, leading to scatter of the test rig’s behaviour.

To examine the scenario, prior parameters of the MBS model have to be defined, which are affected by assumable manufacturing tolerances. In this context, 13 parameters have been identified:

- Masses of all rigid bodies
- Inertias “ $\alpha$ ” of bodies “piston”, “shaft” and “bolt”
- Inertia “ $\gamma$ ” of body “shaft”
- Damping coefficients concerning hydrostatic linear guides “bail-base plate ( $y$ )” and “bail-bolt ( $x, \alpha$ )”
- Damping coefficients concerning friction of hydraulic cylinders (“piston”:  $x/\alpha$ , “bail”:  $y$ )

In the following, the sensitivity of the test rig dynamics on variations of these parameters is examined by sensitivity calculation and MC analyses.

## 3 Sensitivity Calculation

### 3.1 Method of Sensitivity Calculation

The main idea is to pre-evaluate the sensitivity of the test rig performance due to variations of single parameters. Beside information concerning the performance scatter to be expected, promising “adjusting screws” for system optimisation can be derived.

The performance and dynamics of the test rig are analysed using multi-body-simulation. To perform sensitivity calculation, the analytical equations of motion have been set up explicitly using the Lagrange approach. Thus, the system equations are available in symbolic form. The DAE/ODE system of the elastomer test rig is described by 3 equations of motion (1) to (3) with 3 state variables  $x(t)$ ,  $y(t)$  and  $\alpha(t)$  as well as 23 system parameters (13 parameters to be varied, 10 fixed parameters). The system is excited by sinusoidal forces and moments (compare (1) to (3), case 1) as well as noise signals (case 2).

$$3.0e6 \sin(314 t) = cEx1 IW + 4 cEx3 IW^3 + cEx1 X(t) + 6 cEx3 IW^2 X(t) + 3 cEx3 IW X(t)^2 + cEx3 X(t)^3 - 3 cEx3 IW Y(t)^2 - 3 cEx3 X(t) Y(t)^2 - \frac{cEx1 IW^2}{\sqrt{IW^2 - Y(t)^2}} - \frac{4 cEx3 IW^4}{\sqrt{IW^2 - Y(t)^2}} - \frac{6 cEx3 IW^2 X(t)}{\sqrt{IW^2 - Y(t)^2}} \quad (1)$$

$$\begin{aligned} & \frac{3 cEx3 IW^2 X(t)^2}{\sqrt{IW^2 - Y(t)^2}} + \frac{cEx1 Y(t)^2}{\sqrt{IW^2 - Y(t)^2}} + \frac{5 cEx3 IW^2 Y(t)^2}{\sqrt{IW^2 - Y(t)^2}} + \frac{6 cEx3 IW X(t) Y(t)^2}{\sqrt{IW^2 - Y(t)^2}} + \\ & \frac{3 cEx3 X(t)^2 Y(t)^2}{\sqrt{IW^2 - Y(t)^2}} + \frac{cEx3 Y(t)^4}{\sqrt{IW^2 - Y(t)^2}} + (dDx+dEx1+dKx) X'(t) + \frac{(dDx+dEx1) Y(t) Y'(t)}{\sqrt{IW^2 - Y(t)^2}} + \\ & \frac{IW^2(2 mD+mW) Y'(t)^2}{2 \sqrt{IW^2 - Y(t)^2}} + (mD+mK+mW) X'(t) + \frac{(2 mD+mW) Y(t) Y'(t)}{2 \sqrt{IW^2 - Y(t)^2}} \end{aligned} \quad (2)$$

$$3.4e8 \sin(t) = cEy1 Y(t) + \dots + \left( \begin{array}{l} mB+mD + \frac{mW}{4} + \frac{jWz}{IW^2 - Y(t)^2} + \\ Y(t)^2 \left( \frac{4 mD+mW}{4 IW^2 - 4 Y(t)^2} + \frac{jWx IW^2 \tan(AL(t))^2}{(IW^2 - Y(t)^2) \left( \frac{-IW^2}{\cos(AL(t))^2} + Y(t)^2 \right)^2} \right) \end{array} \right) Y''(t) \quad (2)$$

$$1.0e4 \sin(150 t) = (cEa1+cEa3 AL(t)^2) AL(t) + \dots + \frac{jWx IW^2 \tan(AL(t)) Y(t) Y'(t)}{\cos(AL(t))^2 \left( \frac{-IW^2}{\cos(AL(t))^2} + Y(t)^2 \right)^2} \quad (3)$$

The performance and dynamics of the test rig are evaluated by analysing the resulting displacements  $x$  and  $y$  as well as the resulting torsion  $\alpha$  of the body “bolt”, which directly loads the elastomer bushing.

Sensitivity calculation examines the effects of minor parameter deviations from their nominal values for the behaviour of the dynamic systems.

In this case systems are regarded, which are described by differential-algebraic equations (DAEs) of the form

$$F(x, \dot{x}, p, t) = 0 \quad (4)$$

, the equations of motion of the system, whereas  $x(t) \in \mathbb{R}^n$  are state variables and  $p \in \mathbb{R}^m$  summarises parameters of all types, which mean diverse determining factors on developing of the variables.

The factor of interest is the influence, which changes of parameters cause of developing of the variables,

the so-called *sensitivities of parameter*  $\frac{\partial x(t)}{\partial p_i}$  (the

sensitivity of variable  $x$  concerning parameter  $p_i$ ). These sensitivities are computed for the evaluation

of the interesting influence. Sensitivities are functions of time  $t$ . Also these functions can be used as a basis for the determination of derived functions.

By differentiation of the system (4) according to all  $p_i$  the following system can be set up for the computation of first-order parameter sensitivities:

$$\frac{\partial F}{\partial x} \frac{\partial x}{\partial p_i} + \frac{\partial F}{\partial \dot{x}} \frac{\partial \dot{x}}{\partial p_i} = - \frac{\partial F}{\partial p_i} \quad (i=1, \dots, m) \quad (5)$$

(4) together with (5) can currently be solved by Dymola, if (5) is added explicitly. The code DASPK [2] can directly solve the DAE (4) and (5), where (5) is generated within the code automatically.

### 3.2 Results of Sensitivity Calculations

Dymola and DASPK were used to calculate both solution and sensitivities of the elastomer test rig. For the equations of motion the results for  $x$ ,  $y$  and  $\alpha$  were calculated. The solutions from Dymola and DASPK are in accordance. In the following, the influence of parameters on the axial displacement  $x$  and the torsion  $\alpha$  of the bolt are illustrated.

The first-order sensitivities for case 1 are computed using DASPK (figures 8 to 15). The solutions are shown in 8 and 10 (see also figure 5 using Dymola). The figure 9 shows first-order parameter sensitivities of  $x$  regarding parameters of mass. The timeline corresponds to figure 8. Figure 11 where the timeline corresponds to figure 10 shows the first-order parameter sensitivities regarding parameters of inertia.

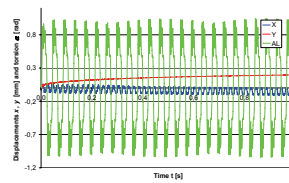


Fig.8: Detail of solution (DASPK)

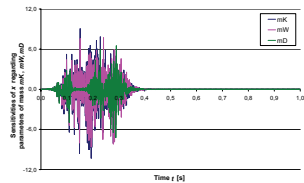


Fig.9: Detail of first-order parameter sensitivities of  $x$  (DASPK)

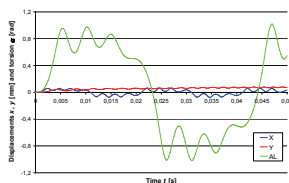


Fig.10: Detail of solution (DASPK)

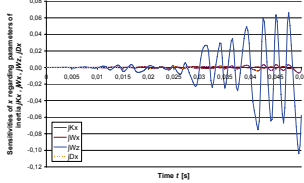


Fig.11: Detail of first-order parameter sensitivities of  $x$  (DASPK)

By classifying the amplitudes of the time-dependent sensitivity functions (e.g. figures, 9, 11, 12, 13, 14, 17 and 18) it can be evaluated, which parameters have a large, a marginal or no influence on the solu-

tion of the system. Parameters with a significant influence to solution  $x$  are the parameters of mass  $mK$ ,  $mW$ ,  $mD$  and the parameters of inertia  $jKx$ ,  $jWz$  and  $jDx$ , see as an example figure 12 and 13. Concerning sensitivities of  $y$ , also parameters of inertia and mass have an influence. Parameters with a significant influence to solution  $\alpha$  are the inertias  $jKx$ ,  $jWz$  and  $jDx$ .

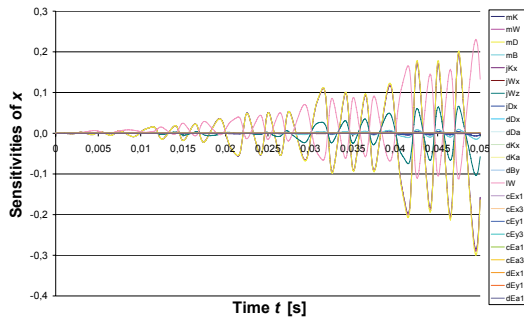


Fig.12: Detail of first-order parameter sensitivities of  $x$  regarding all parameters (DASPK)

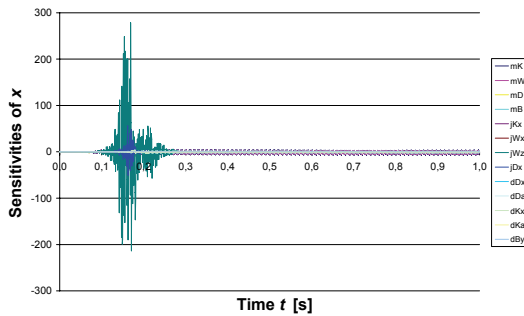


Fig.13: Detail of first-order parameter sensitivities of  $x$  regarding 13 specified parameters (DASPK)

Again figure 14 illustrates the described results exemplarily of parameter of mass  $mK$  regarding  $x$  and  $\alpha$ .

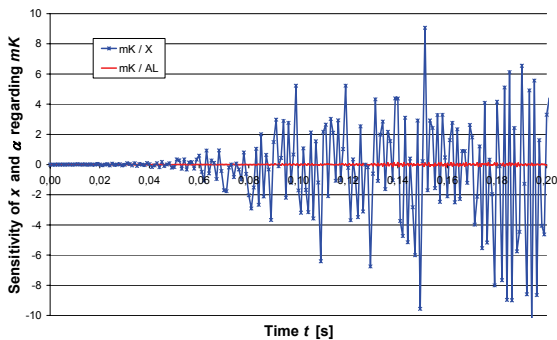


Fig.14: Comparison of influence of parameter of mass  $mK$  on  $x$  and  $\alpha$

Solutions of the original system using different parameter values confirm the sensitivity calculations. Figure 15 illustrates the range of tolerance of solution  $x$  regarding variation of all parameters ( $\pm 0,5\%$  and  $\pm 1\%$ ). The graphic shows that by increasing

time the range of tolerance band is increasing. That is why parameter changing causes not only different amplitudes but also variations in the time behaviour.

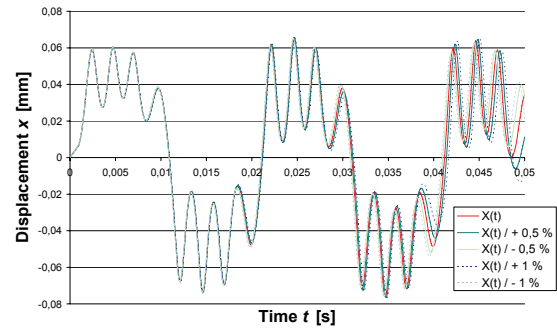


Fig.15: Detail of range of tolerance of solution  $x$  regarding variation of all parameters (DASPK)

In case 2 (figures 17 and 18), sensitivity calculations are carried out by means of Dymola. The resulting first-order parameter sensitivities are approximations by method of difference quotient. This method is explained by a scheme which is illustrated in figure 16. The original model was parallel instantiated with different parameter values, which are necessary for difference quotient calculation.

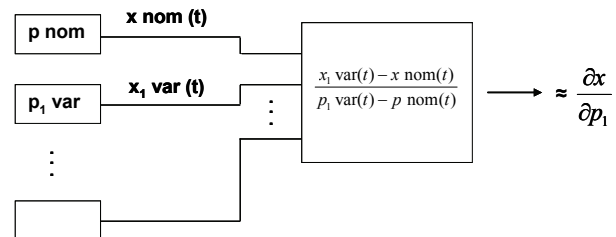


Fig.16: Method of difference quotient using Dymola

Figure 17 shows that all considered parameters have nearly no influence on the solution  $x$  of the system, whereas parameters of mass  $mK$ ,  $mW$  and  $mD$  are dominating parameters.

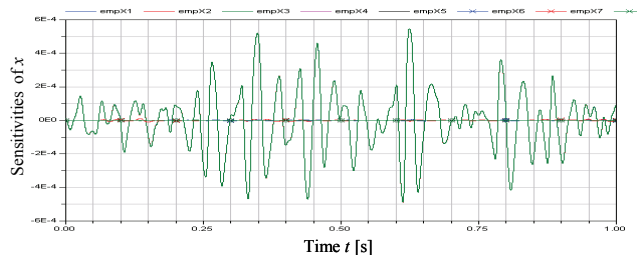


Fig.17: First-order parameter sensitivities of  $x$  for 13 specified parameters (difference quotient)

Parameter with a significant influence on solution  $\alpha$  are parameter of inertia  $jKx$ ,  $jWz$  and  $jDx$ , see figure 18. Regarding case 2, first-order parameter sensitivities will also be carried out by means of DASPK.



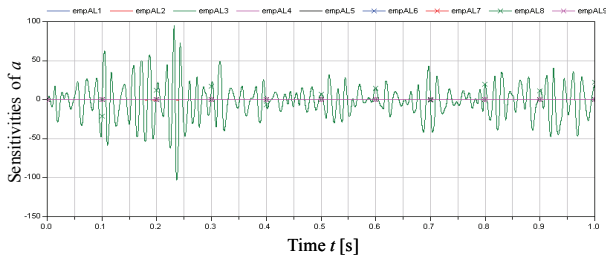


Fig.18: First-order parameter sensitivities of  $\alpha$  for 13 specified parameters (difference quotient)

The results show, that sensitivity calculation leads to comparable results for test case 1 and test case 2.

Related to these sensitivity computations it can be summarised that additionally to the determination of the solution of a concrete DAE system also sensitivity computations are possible. As noted above, the results from Dymola and DASPK are in accordance. Concerning the results of sensitivity calculations, the evaluation of influence of the parameters on the targeted result values is feasible.

## 4 Monte-Carlo Analysis

To evaluate the results of sensitivity calculation MC analyses [3] were performed by usage of both Modelica and MATLAB models.

### 4.1 Modelica Model

Within the Dymola simulator MC simulation is offered as a tool-specific feature. In this paper, another possibility was used which will also be presented as a poster on the Modelica'2008 conference.

In the Modelica model the parameters which are to be varied randomly get their values via a function call. This function, which can be coded as a Modelica function or a C-Function, is parametrised by the user with parameters of the desired random distribution. Repeated Dymola calls (via the scripting language) cause the randomly choice of the value of the chosen parameter. The results of each simulator run have to be collected.

In this case, a uniform distribution with the nominal value 6.0 and the tolerance  $\pm 10\%$  was used as an example, which is specified in the model in this way:

```

model elast
  // parameter of the system
  parameter Real mK = uniform(6,0.10)
  "mass of piston";
  ...

```

Sensitivity calculation described in section 3 yielded an evaluation of system parameters. Then MC analyses for located dominating parameters were determined.

The results of MC analyses for case 1 are illustrated by figures 19 to 23.

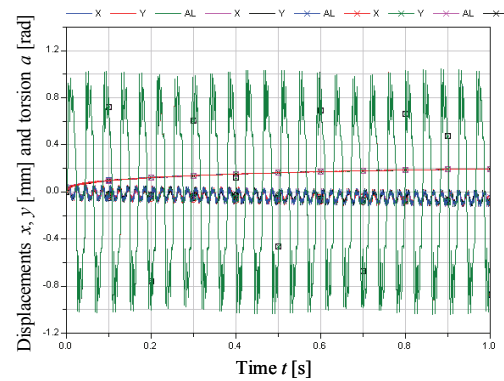


Fig.19: Tolerance band of  $x, y$  and  $\alpha$  regarding parameters of mass

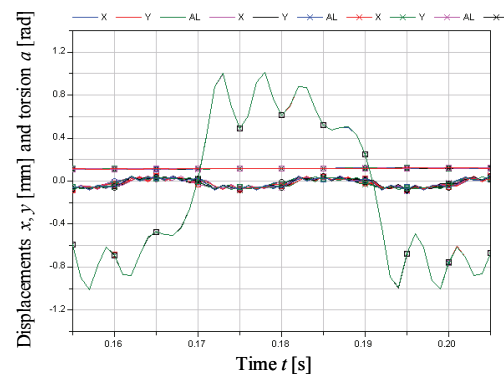


Fig.20: Detail of tolerance band of  $x, y$  and  $\alpha$  regarding parameters of mass

Figure 21 shows that parameters of mass have only a marginal influence on solution  $\alpha$ . These result verified the small tolerance band of  $\alpha$  regarding parameters of mass.

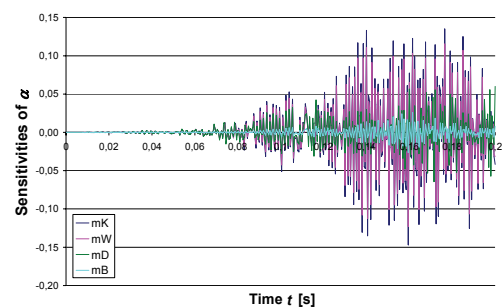


Fig.21: Sensitivity of  $\alpha$  regarding parameters of mass (DASPK)

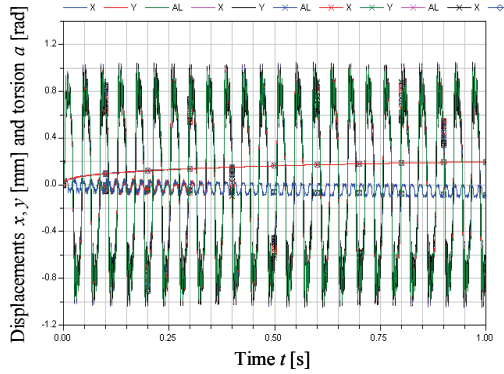


Fig.22: Tolerance band of  $x$ ,  $y$  and  $\alpha$  regarding parameters of inertia

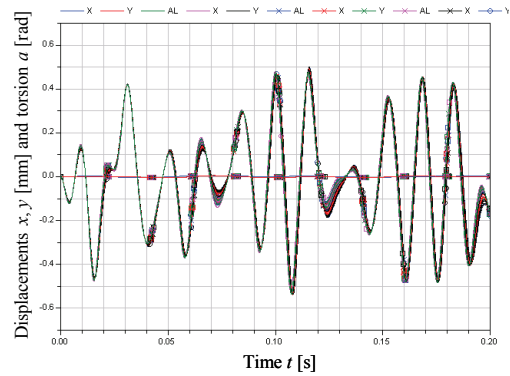


Fig.26: Detail of tolerance band of  $x$ ,  $y$  and  $\alpha$  regarding parameters of inertia

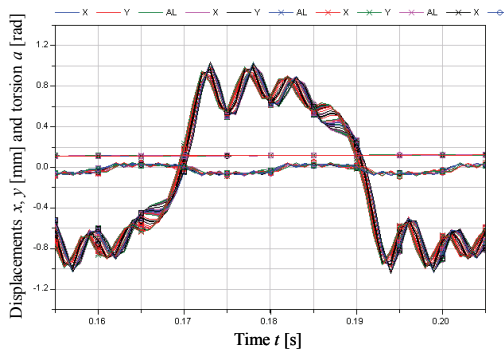


Fig.23: Detail of tolerance band of  $x$ ,  $y$  and  $\alpha$  regarding parameters of inertia

MC analyses verified the results of sensitivity calculation, see section 3.

#### 4.2 MATLAB Model

The sensitivity analysis of the MATLAB model was performed with the Fraunhofer LBF inhouse software MASIMO. The software creates sample sets of user-defined parameters based on Latin-Hypercube-Sampling methods and automatically performs the needed simulations in MATLAB. MASIMO was, among other things, applied during the EC funded project “MODBOGIE” [4] to perform sensitivity analysis of a complex locomotive model.

The MC analyses each contained 100 simulations for test case 1 and test case 2. All 13 parameters (cp. ch. 2.2) were set to vary in a range of  $\pm 10\%$  of their nominal value, while an equal distribution of parameter values was defined, respectively.

To analyse the resulting time series of the simulations, scalar evaluation quantities  $x_m$ ,  $y_m$  and  $\alpha_m$  were defined, taking the arithmetic mean value of the amount of displacements and torsions  $x(t)$ ,  $y(t)$  and  $\alpha(t)$ . Following, the parameter  $x_m$  is examined, exemplarily.

Figure 27 shows an qualitative Anthill plot of  $x_m$  as a function of the piston mass for test case 1. Each point represents the (converted) result of one single simulation of the MC analysis. The diagram shows the trend, that an increasing piston mass leads to decreasing values of  $x_m$ . In general, Anthill plots can be used to get a first impression of sensitivities and trends.

The results for case 2 are shown by figures 24 to 26.

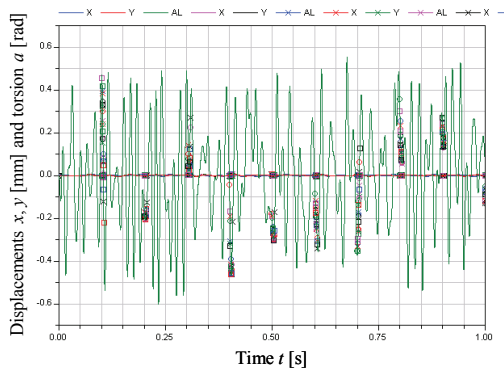


Fig.24: Tolerance band of  $x$ ,  $y$  and  $\alpha$  regarding parameters of mass

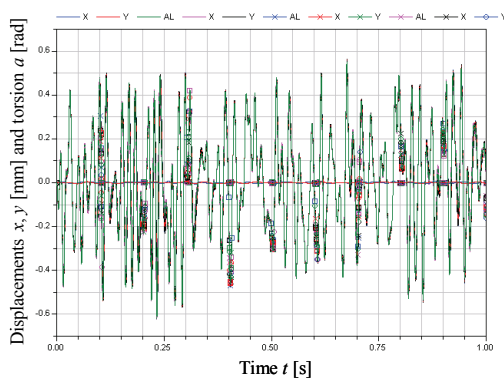
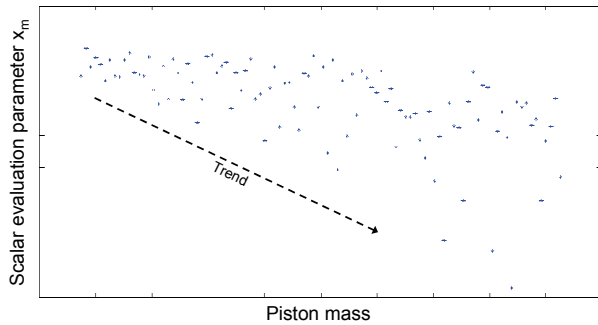
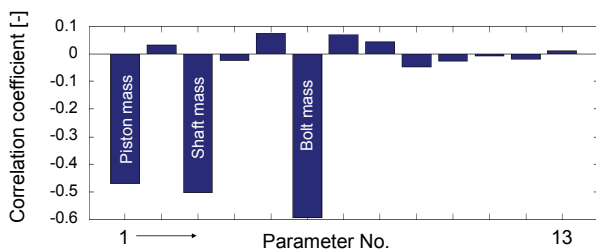


Fig.25: Tolerance band of  $x$ ,  $y$  and  $\alpha$  regarding parameters of inertia

Figure 27: Anthill-plot of  $x_m$  as a function of piston mass

To derive further information, correlation coefficients between the result values  $x_m$ ,  $y_m$  and  $\alpha_m$  and the varied input parameters can be applied. Figure 28 exemplary shows the correlation between  $x_m$  and the input parameters. In this context, negative correlation coefficients point out, that an increasing parameter value leads to decreasing result quantities. It is obvious, that the masses of piston, shaft and bolt affect  $x_m$  dominantly.

Figure 28: Correlation of  $x_m$  with the varied parameters (1...13), case 1

Similar investigations were performed for the result values  $x_m$  and  $\alpha_m$ . The analysis of  $\alpha_m$  showed prior sensitivities on the inertias of piston, shaft and bolt, while  $y_m$  is dominantly affected by the masses of all 4 bodies.

The analysis of test case 2 led to comparable results. Again, the body masses of piston, shaft and bolt affect  $x_m$ , while the torsion  $\alpha_m$  is dominantly influenced by the inertias of these bodies.  $y_m$  again is dominantly affected by the masses of all 4 bodies.

## 5 Conclusions

In this paper, the application of sensitivity calculation was presented by example of a multi-body simulation model of an elastomer test rig. The results show, that sensitivity calculation has the potential to pre-evaluate prior parameters of a model, which ex-

emplarily can be deeper analysed by a following MC analyses. An indispensable precondition for applying sensitivity calculation is the provision of the equations of motion in a symbolic representation.

Especially for complex models with a high number of DOF or long periods to be computed, the preselection of parameters can lead to a significant reduction of computational effort. Even in case of the test rig example, which only comprises 4 DOF and rather small time series to be computed (< 2 seconds), each simulation of the MC analysis took approximately 2 minutes (Pentium 4, 3 GHz). Resulting, a complete analysis with 100 simulations and 13 parameters took more than 3 hours.

A sensitivity calculation using DASPDK respectively 13 parameters (until  $t_{\text{end}}$  1 s) took approximately 10 minutes. Using Dymola a sensitivity calculation (as shown in figure 16) took maximal approximately 40 minutes and a MC analysis with 50 simulations and 4 varied parameters maximal approximately 12 minutes. Resulting, a complete MC analysis using Dymola with 100 simulations and 13 parameters (2 seconds) would take approximately 2.5 hours. These computation times point out, that sensitivity calculation is able to reduce effort considerably.

Within the Dymola simulator, MC simulation is offered as a tool-specific feature. In this paper a more common possibility was presented, which describes the MC method on the Modelica language totally. Both methods are very time consuming. Using Dymola the effects of parameter tolerances can be calculated by MC simulation (with a high computational effort), using the sensitivity system (5), which has to be added manually or using the finite difference approximation (see figure 16).

Using the code DASPDK, system (5) is generated within the code automatically. It would be desirable, if this possibility (and also regarding a similar system for second-order parameter sensitivity) would be existant also in Dymola. So far, an operator of sensitivities like the existing operator  $der()$ , the derivation with respect to time, is absent.

Note, that the solver DASPDK allows the computation of first-order parameter sensitivities. The interpretation of these results leads to a classification of the importance of the system parameters regarding the effect to the variables.

## 6 Outlook

The next steps will cover following topics:



## Parameter Sensitivities of Second-Order using DASPCK

In this section a short description is given to determine second-order parameter sensitivities by means of DASPCK. By means of differentiation of the system (5) according to all  $p_i$  second-order parameter sensitivities are computed. As mentioned before, the code DASPCK can solve the DAE (4) and (5). By differentiation of the system (4) according to all  $p_i$  and using of this system as a new system (4) in the source code, the second-order parameter sensitivities are generated automatically.

Another way to determine the second-order parameter sensitivities is the modification of source code of DASPCK. Therefore, an aim is to extend the source code of DASPCK to generate the system of second-order parameter sensitivities automatically by differentiation of system (5). Then DASPCK could solve (4), (5) and also the system of second-order, where (5) and the system of second-order could be generated within the code automatically.

## Introduction of scalar evaluation quantities

In continuative work, analyses concerning scalar evaluation quantities, which are derived from the results in the time domain, are planned. Examples for these scalar evaluation result quantities are the first eigenfrequency or the mean value of the amplitude spectrum in a defined frequency range. The first eigenfrequency can be computed in MATLAB directly from the condition matrix of the elastomer test rig model. Using Dymola, the condition matrix can be generated and denoted by linearisation of the original model. Within Modelica there are also matrix functions, which are useful for this context. The first eigenfrequency can be derived from simulations in the time domain and a following Fourier transformation. For each DOF  $x$ ,  $y$  and  $\alpha$ , then a scalar quantity can be calculated.

Regarding sensitivity calculation, problems can occur in this context, because scalar evaluation quantities are not directly available in the DAE system. If the evaluation quantity can be calculated during the simulation, sensitivities are automatically present. Otherwise, derived evaluation variables have to be calculated by post-processing. This challenge will be discussed in further publications.

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