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Modularised Tyre Modelling in Modelica

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Abstract

Good tyre models are essential for driving simulation of all ground vehicles using pneumatic tyres. However, tyre behaviour is extremely complex, often requiring different models for the various behaviours. This paper presents an implemented tyre model library that takes advantage of the modular modelling possibilities in Modelica to combine different models. For example, the different sub models representing tyre-to-road contact can be combined with various models for tyre belt behaviour. The library can be used together with vehicle models in one, two and three dimensions.



Figure 1: *The tyre construction of a normal passenger car tyre. 1 - tread, 2 - sidewall, 3 - radial cord, 7 - belt, 9 - bead.*

1 Introduction

Complex tyre behaviour is a direct result of the tyre construction. While looking much like a simple rubber doughnut attached to the rim, the tyre construction is vastly more complex, figure 1.

The two main functions of the tyre, is force generation in the road plane and suspension of the vehicle mass. The force generation is made possible by the rubber tread, causing a high friction coefficient with the road surface. The suspension of the vehicle mass is managed by the carcass, consisting of the belt, radial cords and beads. The radial cords work like the spokes of a bicycle rim; a pre-tension must be exerted by pressurised air inside the tyre to carry the loads. Because of the two main functions of the tyre, modelling of the behaviour can be split into modelling the functions of the tyre, rather than modelling the tyre itself.

Naturally, there are more areas to take into account when the combination of components for the tyre is to be selected and calculated. Areas such as comfort, traction and cornering are covered by

the main functions. There are also issues such as rolling radius, steering behaviour, force feed back, vibrations, sound radiation, water dispersion, directional stability to be considered when designing a tyre.

Depending on the type of modelling required, single, several or all areas involved need to have exclusive models allocated. The selection of the areas of interest are made by the modeller. Therefore, it can be of great help to possess a flexible way to add and subtract function models of the tyre.

When examining the main functions, the demand made on precision, processing time and comprehension, generates a need for different models for the same function. The suspension function is mostly modelled dynamically and is used for comfort purposes. A simple model could

be a spring - mass - damper system with one degree of freedom. If more complexity is required, a rigid or flexible belt model connected with pre-tensioned springs and dampers to the rim with correct geometry as presented by Böhm [1] will give a fairly good representation, figure 2. The most complex models are the full FE-models with anisotropic cord, internal rubber damping, complex friction behaviour and temperature calculation.

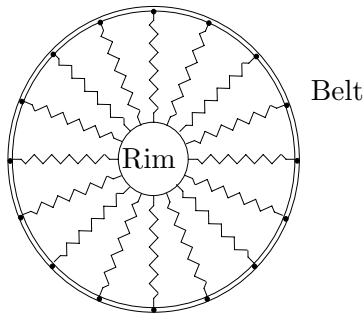


Figure 2: *Rigid belt model with pre-tensioned springs.*

For lateral and longitudinal direction, force generation models for cornering and traction are often empirical models, where measured data are fitted to polynomials to give an analytical function, figure 3, however, physical models also exist. A simple modelling approach is the use of a cornering stiffness; a linear relation between force and slip, valid only in a very limited area of handling. A simple but wider model would be a curve fit of the forces as a function of slip angle and slip ratio with saturation and load dependency. A more complex models is the Magic Formula [2], developed by TU Delft and Volvo Cars Corporation. This model also includes force due to camber, rolling radius changes and rolling resistance. The Magic Formula will give lateral and longitudinal force, aligning moment and driving moment from various analytical functions. All these models assume point contact and do not take into account the width or length of the contact patch. This assumption simplifies the calculations and reduces the need for accurate road data.

2 Definitions

Because of the tyre complexity, several reference frames are necessary to model its behaviour. To

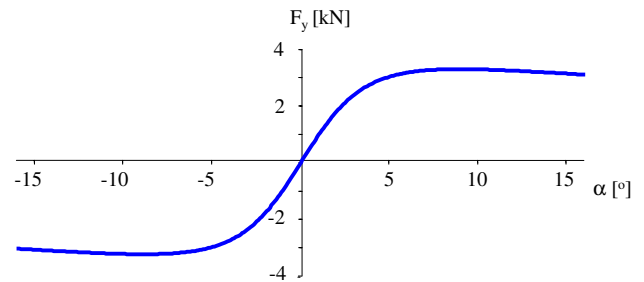


Figure 3: *Typical appearance for the lateral force, F_y , as function of slip angle, α , in this case generated with the Magic Formula, [3].*

ensure that the model structure allows the simple addition and reuse of components within new models, the modelling is based on two international standards; DIN and TYDEX. According to the DIN coordinate representation, illustrated in figure 4, x points forward, y to the left and z right up. The TYDEX definition of the carrier frame, C , and contact frame, W , is shown in figure 5. The carrier frame is fixed at the car's suspension and the contact frame is located at the intersection of the carrier frame's z -axis and the road plane. Generally, the contact point is not located at the origin of frame W and therefore a frame W' is introduced. Frame R , located at the rim, is used since the forces and torques generated by the tyre cannot act directly at the carrier.

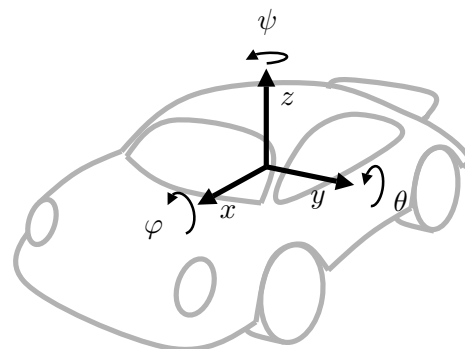


Figure 4: *The coordinates according to DIN. Yaw, pitch and roll are indicated with ψ , θ and φ , respectively.*

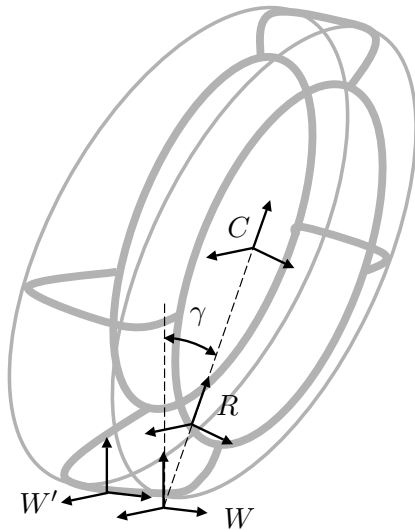


Figure 5: The frames used in the modelling. The deformation of the tyre belt results in a displacement of the contact point from W to W' . The carrier frame, C , and the rim frame, R , do not rotate with the wheel.

3 Model structure

The models that this modularisation is based on all assumes that the tyre-road contact patch can be approximated by a point. This assumption is used to define the cuts at the tyre-road, the tyre rim and the rim-carrier. With this modularisation, a tyre can be described with three parts, hub, tyre belt and tyre-road contact, as shown in figure 6. These components are described in more detail below, the frames referred to are indicated in figure 5.

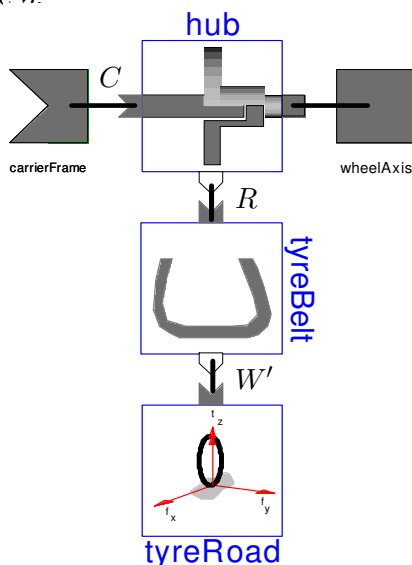


Figure 6: The tyre models consists of three main subcomponents: The hub, the tyre belt and the tyre-road contact.

3.1 Hub

The hub is mounted on the carrier and allows the wheel and drive axis to rotate around the y -axis. Since none of the frames rotates with the wheel, the Modelica-model of the hub transforms the forces and torques acting at the rim frame, R , to the carrier frame, C and the drive axis. This cannot be done by a standard component since the torque around the y -axis only should act at the drive axis.

3.2 Tyre belt

The tyre belt carries the load on the wheel distributing the forces from the tyre-road contact patch to the rim. As mentioned earlier, the patch is approximated with a point. The Modelica-model of the tyre belt describes the relation between the contact point frame, W' , and the rim frame, R . There are several ways of implementing this where the simplest rim model neglects the "horizontal" deformation of the tyre belt which gives $W' = W$.

3.3 Tyre-road contact

The tyre-road contact generates friction forces and torques that act on the tyre belt. The Modelica model calculates and applies these to the contact point frame, W' . Most of these models are empirical and generates forces as functions of slip, camber angle etc. An example is the Magic Formula model used in figure 3.

4 Utilities

In addition to the main components described in section 3.1-3.3, other components are realised to simplify the extension of the `Wheels` library. Some of which are described in more detail below.

4.1 State handler

This component is introduced since the tyre models are not strictly physical. This is due to the modularisation and to the fact that some tyre states cannot be calculated from the simpler models. For example, $x-y$ vehicle models, as the bicycle model [3], require that the normal force and the camber angle are given as parameters. More detailed models, where this information is required,

can be used in a convenient way by using this state handler.

4.2 Contact point

The `ContactPoint` component is used within the `TyreBelt` to calculate frame W . Depending on the implementation this can be implemented in several ways. As described in [4], external road information can be provided via the `inner/outer` Modelica constructs. Information needed to calculate W is the road normal and altitude as a function of longitude and latitude.

4.3 Camber angle

The camber angle is the angle between the road normal and the z -axis of the carrier frame, indicated with γ in figure 5. The implementation in Modelica is more general and calculates the angle between the z -axes of any pair of frames.

4.4 Tyre radius

There are at least three different tyre radii, undeformed radius, R , effective rolling radius, R_e and loaded radius, R_l . The loaded radius is the wheel centre's height over ground, while the effective rolling radius is a measure of how far a free rolling tyre travels per revolution. Generally, $R_l < R_e < R$. This means that the rolling tyre travels farther per revolution than determined by using the wheel's centre height as rolling radius. Mixing these definitions would thus lead to erroneous simulation results.

In the `Wheels` library, the loaded radius, can be calculated at any time, while the effective rolling radius as well as the undeformed radius must be given as parameters or functions.

5 Interfaces

As mentioned earlier, the `Wheels` library can be used in one, two and three dimensions. The interfaces for these are taken from the `Modelica.Mechanics.Translational` (1D) and `ModelicaAdditions.MultiBody` (3D), [5]. To handle the the two-dimensional models, a library `MultiBody2D` has been realised. Additionally, for example, the interfaces from `Modelica.Mechanics.Rotational` are used for the wheel

axis and thus, the wheels can be connected with available packages for power-trains.

To guarantee that the different tyre models are replaceable, they all are based on interface models, `BaseWheel[*]`, where $[*]$ is the dimensions they are meant for, e.g. `BaseWheelXY` for a wheel in the XY plane. In total, seven different interfaces are realisable, X, Y, Z, XY, XZ, YZ and XYZ.

5.1 MultiBody2D

This library is used to describe two-dimensional mechanical systems and is based on the same ideas as the three dimensional `ModelicaAdditions.MultiBody` library, [5]. Instead of three DOF each for translation and rotation as in the 3D-case, this library has two DOF for translation and one DOF for rotation, allowing only planar motion. The Modelica definition of the interface is:

```
connector Frame_a
  Modelica.SIunits.Position r0[2];
  Real S[2, 2];
  Modelica.SIunits.Velocity v[2];
  Modelica.SIunits.AngularVelocity w;
  Modelica.SIunits.Acceleration a[2];
  Modelica.SIunits.AngularAcceleration z;
  flow Modelica.SIunits.Force f[2];
  flow Modelica.SIunits.Torque t;
end Frame_a;
```

In figure 7, parts of the `MultiBody2D`-library are shown.

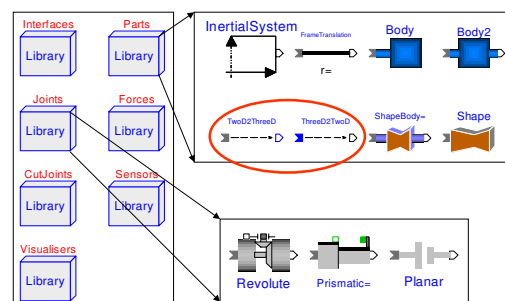


Figure 7: The `MultiBody2D` library. The parts indicated provide an interface to the `ModelicaAdditions.Multibody` library.

6 Usage

To get a better idea of how the `Wheels` library can be used, some examples and aspects are presented below.

6.1 Bicycle model

The bicycle model is a very simple model of a vehicle. However, it provides essential information about a vehicle's behaviour. The motion is restricted to the x - y plane and the, normally, two wheel per axis are replaced by one single wheel, i.e. a bicycle with the centre of gravity in the ground plane.

In figure 8, a bicycle model of a car with a trailer is shown. The bicycle is connected to an inertial system (1) via a planar joint (2) allowing x - y translation and rotation around z . The bicycle consist of a body (3) with mass and inertia, translations (4,5) from the centre of gravity to the front and rear wheels and a revolute joint for the steering (6). The wheels (7,8) could be of any kind as long as they use the `BaseWheelXY` interface. Additionally, a connector COG (9) makes it possible to attach further components to the model. In this case a trailer is attached, it would also be possible to add, for example, aero dynamical models and sensors.

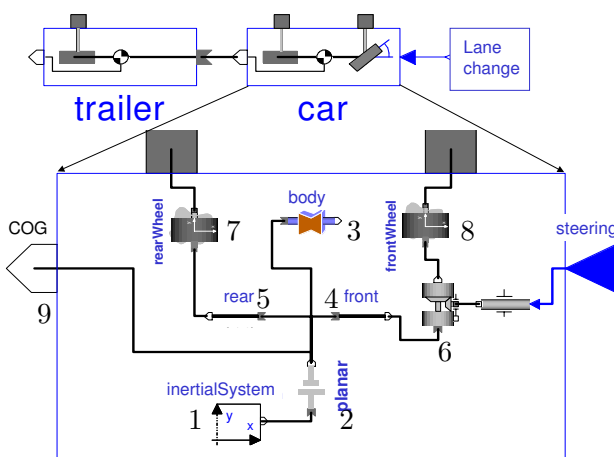


Figure 8: An implementation of a bicycle model with the `MultiBody2D` library, note how simple it is to add a trailer.

6.2 Switching tyre models

Modelica provides a convenient way of replacing models with the same interface using

`replaceable` and `redeclare`. To illustrate how this works, an example with a replaceable front wheel in a bicycle model in the x - y plane is discussed.

Since all wheel models for the x - y plane have the same interface, `BaseWheelXY`, this is used within the declaration of the bicycle model:

```
model Bicycle
  replaceable BaseWheelXY frontWheel;
  ...
end Bicycle;
```

Whenever a bicycle model is used, the tyre model can easily be replaced with any x - y model:

```
model AnyXY
  extends BaseWheelXY;
  ...
end AnyXY;
```

```
model BicycleTest
  Bicycle bicycle(redeclare
    AnyXY frontWheel);
  ...
end BicycleTest;
```

Without this feature, a new model, in this case the `Bicycle` model, would have to be realised for each combination of wheel models studied.

6.3 Parameterisation

The more complex a tyre model gets, the more parameters are required. As an example, the Magic Formula model to calculate the steady-state forces and torques acting at the contact point requires 73 parameters. Since the parameter sets differ from model to model, it would be lengthy to define each parameter value within a `redeclare` phrase as described in section 6.2.

Instead, the parameter sets are gathered in records that are used like structural variables. These records can then be treated as single parameters and be stored as models in the library.

6.4 Adding components for visualisation

Due to the modularisation, additional components can easily be attached. To illustrate this, an example where animation information is added to the tyre is shown. In figure 9, components named `contact` and `contactPatch` are added. `contact`

provides visual information about the forces as vector shapes acting at the cut. In this case, the

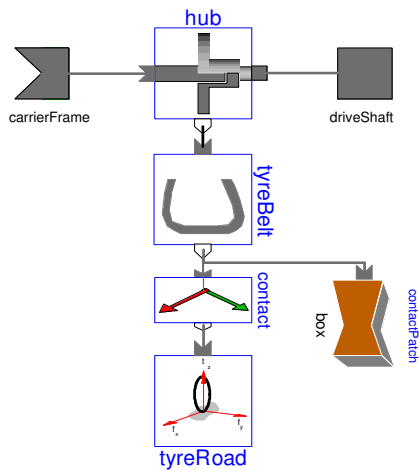


Figure 9: Since the components are separated it is easy to add additional information, in this case graphical information.

contact point forces are visualised but this component could of course be attached at any cut to illustrate forces. `contactPatch` is an ordinary box shape that visualises the contact point as a square. This is a very simple way to to illustrate the tyre belt deformation. Figure 10 shows a snapshot of the animation result.

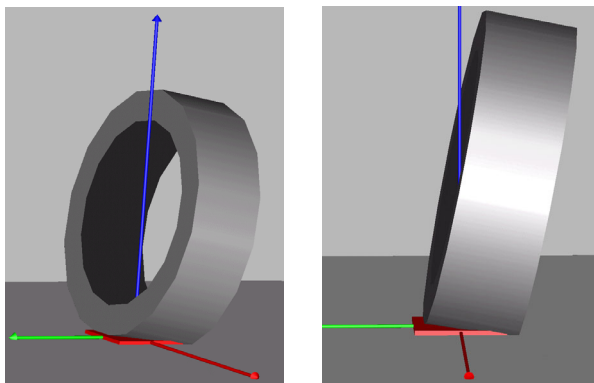


Figure 10: These animation pictures shows forces in x -, y - and z - direction of the W' frame. The tyre belt deformation is indicated by the red square.

6.5 Mixing dimensions

Up to this point it has been assumed that the tyre model used has the same dimensions as the car model that is used. However this is not a necessity as will be shown in the following example.

Assume that the y - z motion of a car is to be studied, when driving with constant speed on a highway, and that an XYZ model is to be used. With the component `TwoD2ThreeD`, a connection between the tyre model in 3D and the car in 2D can be made as illustrated in figure 11. The additional information needed for the 3D frame, such as longitudinal speed, is given as parameters.

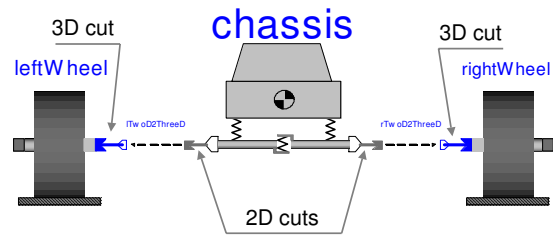


Figure 11: An example of how dimensions can be mixed. The `TwoD2ThreeD` component, allows a 3D wheel to be connected to a 2D car model.

6.6 Choosing accurate models

Even if the modularisation eases the modelling work, the need to understand the models, in order to know their limitations is still a necessity. Performing manoeuvres that takes the tyre model outside the valid range may have fatal consequences as seen in the following example:

With a vehicle model, two steering step inputs of 0.03 rad and 0.2 rad at the front wheels are simulated. First, a linear tyre-road contact model is used and then, the same simulations are performed with a Magic Formula tyre-road contact model. In the first case, the wheel slip is low and both models behave similarly, but for the second case, the linear model is no longer relevant, figure 12.

In the above example, the limitation of the linear model is deliberately made obvious, but often it is harder to detect when the range of the model's validity is exceeded. To help the user avoid these situations, the plan is to add an alert procedure that checks crucial variables against model limitations. This function is not yet implemented.

6.7 Case study

To give an example of the usage of the `Wheels` library and further illustrate the necessity of cor-

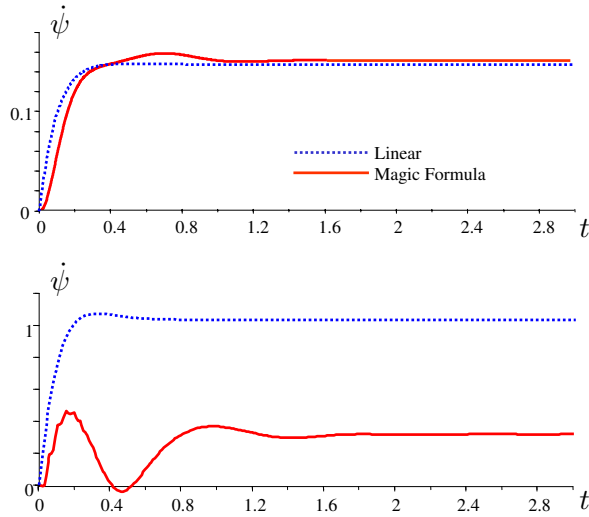


Figure 12: The plots show the yaw speed response. The upper plot shows similar results for both models while the lower plot shows how the linear model lacks ability to predict skidding. Obviously, this manoeuvre is outside the model’s range of validity.

rect tyre models, a parameter study is performed. The concepts *understeer* and *oversteer* are familiar to many and a widely spread opinion is that a car with more ”weight on the front wheels” will understeer while more ”weight on the rear wheels” will make the car oversteer.

To show the limitation of this theory a variation of the location of the centre of gravity is compared to a variation of the tyres’ maximum side force. The results are shown with the *understeer gradient*, K_{us} , which is a quantification of *how much more one has to turn the steering wheel to increase the lateral acceleration, compared to a neutral car*. Mathematically this is defined as

$$K_{us} = \frac{1}{i_s} \frac{\partial \delta_H}{\partial a_y} - \frac{\partial \delta_A}{\partial a_y} \quad (1)$$

where δ_H is the steering wheel angle and i_s the ratio between the steering wheel angle and the wheel angle. δ_A , called the *Ackermann angle*, is the wheel angle required to turn the car at low speed with no slip and a_y is the lateral acceleration.

Positive K_{us} thus means that the driver has to turn the steering wheel more to get the desired motion compared to driving a neutral car, i.e. the car understeers. If K_{us} is negative, less input has to be given and the car oversteers.

To perform the variation, the variables λ and μ

are introduced. λ indicates the distance from the front wheels to the centre of gravity. μ scales the height of the lateral force peak, shown at about $\alpha = 5^\circ$ in figure 3. Thus $\mu = 1$ does not affect the tyre and $\lambda = 0.5$ means that the centre of gravity is located half way between the wheel axes. The variation is done according to:

| | | |
|-------------|----------------------|--------------------|
| λ^0 | $\lambda = 0.50$ | |
| λ^- | $\lambda = 0.45$ | |
| λ^+ | $\lambda = 0.55$ | |
| μ^0 | $\mu_{front} = 1.0,$ | $\mu_{rear} = 1.0$ |
| μ^- | $\mu_{front} = 0.9,$ | $\mu_{rear} = 1.1$ |
| μ^+ | $\mu_{front} = 1.1,$ | $\mu_{rear} = 0.9$ |

The simulation is made using a bicycle model at constant speed. The steering wheel input is given as series of step increase. Between each step, the car stabilises and the lateral acceleration is logged. This information is then used to calculate K_{us} .

The result of the simulations is shown in figure 13. At low lateral accelerations, K_{us} depends solely on λ and the theory presented above holds. As a_y increases, μ gets more important and for high a_y , when the car skids, the tyre choice dominates. Neglecting this effect when designing a car would thus lead to unknown behaviour in critical situations.

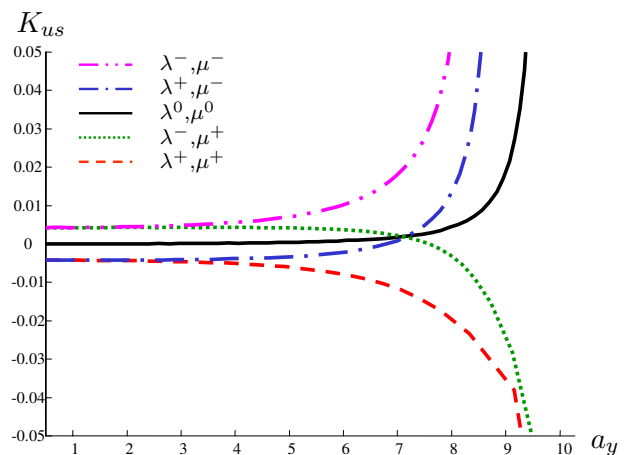


Figure 13: Result of the parameter variation.

7 Limitations

The largest limitation of the modularisation is that the selection of model accuracy must be problem based. This will require complete understanding of the problem by the user to be able to use the appropriate models. Another limit of the modularisation presented, is that the contact patch is assumed to be a point. This assumption is not valid for situations such as; driving on uneven surfaces, hitting pavements or crossing potholes. Additionally, when the steering input changes rapidly compared to the speed, for example when parking, the slip cannot be assumed to be constant over the whole patch which makes it much harder to use a contact point approximation.

8 Conclusions

The suggested division of the tyre model into hub, belt and road contact has simplified the reuse of model components and thus, made it easy to extend the `Wheels` library and to add new functionality. At the same time the coupling of behaviour that are functions of two or more parts is more complicated than without the separation.

The possibility to quickly replace the tyre models or sub-components when switching model accuracy will be of great advantage and this possibility will most certainly be used more frequently as the selection of components grows.

9 Acknowledgements

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