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Modeling and Simulating the Efficiency of Gearboxes and of Planetary Gearboxes

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Abstract

It is shown how to model and simulate frictional effects present in gearboxes and in planetary gearboxes. This includes modeling of gear wheel stucking and sliding due to Coulomb friction between the gear teeth leading to load torque dependent losses. This allows reliable simulation of, e. g., stick-slip effects in servo drives or gear shifts in automatic gearboxes. It is also discussed how the friction characteristics can be measured in a useful way. The presented models are implemented in *Modelica* and demonstrated at hand of the simulation of an automatic gearbox.

1 Introduction

Gearbox dynamics due to friction, elasticity and backlash in the gear has often a strong impact on the performance of the system in which the gearbox is contained, such as for robots, machine tools, vehicles, power trains. It is both difficult to simulate gearbox effects and to get reasonable agreement between measurements and dynamic simulations.

In the current *Modelica* standard library `Modelica.Mechanics.Rotational` [5] several model components are available to simulate gearbox effects, especially bearing friction. *Missing* is the satisfactory handling of *mesh efficiency* due to friction between the teeth of gear wheels which leads to load torque dependent losses. In this article it is shown in detail how this problem can be solved for *any* gearbox that has *two* or *three external shafts*, i. e., standard gears and a large class of planetary gears.

Before going into the details of an appropriate gear efficiency model, it is important to analyse the different frictional effects in a gearbox. Friction is present between two surfaces which slide on each other. In a gearbox, this occurs in the gear bearings and between the gear teeth which are in contact to each other. The effect of these two cases is quite different.

1.1 Bearing Friction

The torques acting at a bearing are shown in figure 1. The shaft in the bearing has the torques τ_A and τ_B on the two sides. Losses due to friction are described by the additional torque τ_{bf}. Torque equilibrium yields

$$\tau_B = \tau_A - \tau_{bf} \ . \qquad (1)$$

where

$$\tau_{bf} = \begin{cases} \geq 0: & \omega > 0 \\ \leq 0: & \omega < 0 \\ \text{so that } \dot{\omega} = 0: & \omega = 0 \end{cases} \qquad (2)$$

The friction torque τ_{bf} is essentially a function of the shaft speed ω, the bearing load f_N (=force perpendicular to bearing axis), the bearing temperature T, the bearing construction and the used lubrication (for more details, see, e. g. [6]). Since the bearing load is usually constant (but not zero) and independent of the gearbox load torque, and all other factors can be often regarded as constant for certain operation conditions, the bearing friction is essentially a function of the relative speed, $\tau_{bf}(\omega)$, and has a characteristic as given in figure 2.

If $\omega \neq 0$ the friction torque τ_{bf} is computed from the sliding friction characteristic according to figure 2. If $\omega = 0$ the bearing is stuck due to the bearing load in combination with Coulomb friction, and therefore the friction torque τ_{bf} is an unknown constraint torque which is computed so that $\dot{\omega}$ vanishes. How to model and simulate this effect is described in detail, e. g., in [8].

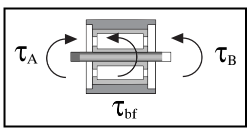

Figure 1: Torques at a bearing

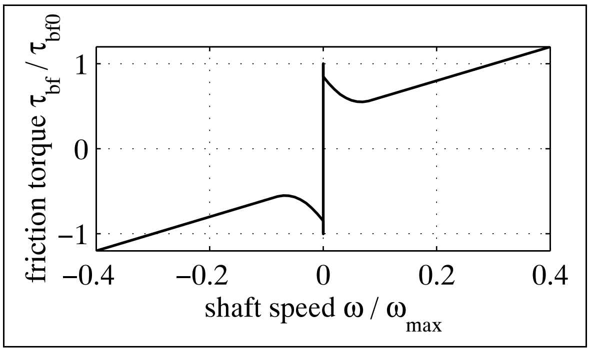

Figure 2: Typical bearing friction characteristic

1.2 Mesh Friction

In figure 3, two gear teeth in contact are shown. In order that the teeth neither penetrate nor separate, the normal velocities in contact point C need to be identical and therefore the tangential velocities are different, i. e., the teeth slide on each other, see, e. g., [3] (the tangential velocities are only identical, if $\omega_A = 0$ or if point C is at W, see figure 3). As a result, in contact point C Coulomb friction $f_R = s_v \mu f_N$ is present, where f_R is the friction force in the contact plane, f_N is the force perpendicular to the contact plane, $s_v = \pm 1$ depending on whether C is below or above pitch circle r_A and $\mu = \mu(v_{rel}, T)$ is the sliding friction coefficient which is essentially a function of the relative velocity v_{rel} between the contact planes and the temperature T at the contact point. Note, that the contact planes may become stuck to each other if $v_{rel} = 0$, i. e., if $\omega_A = 0$. Then, f_R is a constraint force calculated from the condition that $\dot{\omega}_A = 0$. Torque equilibrium in figure 3 yields

$$\begin{aligned} 0 &= \tau_A - f_N l_A + f_R d_A \\ 0 &= \tau_B - f_N l_B + f_R d_B \\ f_R &= s_v \mu f_N \end{aligned} \quad \Rightarrow \tau_B = \frac{l_B \left(1 - s_v \mu \frac{d_B}{l_B}\right)}{l_A \left(1 - s_v \mu \frac{d_A}{l_A}\right)} \tau_A \,.$$

Utilizing teeth contact geometry and gear ratio i

$$\cos\alpha = \frac{l_A}{r_A} = \frac{l_B}{r_B} \qquad \Rightarrow \qquad \frac{l_B}{l_A} = \frac{r_B}{r_A} = i$$

results in

$$\tau_B = i \eta_{\text{mf1}} \tau_A \quad \text{with} \quad \eta_{\text{mf1}} := \frac{1 - s_v \mu \frac{d_B}{l_B}}{1 - s_v \mu \frac{d_A}{l_A}} \,. \qquad (3)$$

Since $d_A/l_A < d_B/l_B$ for $s_v = +1$, $d_A/l_A > d_B/l_B$ for $s_v = -1$, and $0 \le \mu < 1$, it follows from (3) that η_{mf1} is between 0 and 1.

In a similiar way it can be shown (see, e. g., [7]) that the relative velocity v_{rel} is calculated as $v_{rel} = k(\cdot)\,\omega_A$ where $k(\cdot)$ is a function of the geometric quantities d_A, d_B, l_A, l_B. Since all these quantities can be computed from gear-wheel constants and the absolute angle φ_A of shaft A, $\mu = \mu(\varphi_A, \omega_A, T)$ and therefore $\eta_{\text{mf1}} = \eta_{\text{mf1}}(\varphi_A, \omega_A, T)$.

The derivation above is only valid if $\omega_A > 0$, because the friction force at the driven tooth is always directed in opposite direction to the relative sliding velocity. If $\omega_A < 0$ the sign of the friction force changes, yielding:

$$\eta_{\text{mf2}} \tau_B = i \tau_A \quad \text{with} \quad \eta_{\text{mf2}} := \frac{1 + s_v \mu \frac{d_A}{l_A}}{1 + s_v \mu \frac{d_B}{l_B}} \,. \qquad (4)$$

The derivations assume that the ”right” side of tooth edge A is in contact. If the ”left” side is in contact, the sign of the normal force changes. Collecting everything together and neglecting the temperature and position dependency of $\eta_{\text{mf1}}(\cdot), \eta_{\text{mf2}}(\cdot)$ finally results in the basic formula for mesh friction:

$$\hat{\eta}_{\text{mf}} := \begin{cases} \eta_{\text{mf1}}(|\omega_A|) & : \begin{cases} \tau_A \,\omega_A > 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A > 0 \end{cases} \\ 1/\eta_{\text{mf2}}(|\omega_A|) & : \begin{cases} \tau_A \,\omega_A < 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A < 0 \end{cases} \\ \text{so that } \dot{\omega}_A = 0\text{:} & \omega_A = 0 \end{cases} \qquad (5)$$

where $\eta_{\text{mf1}}(|\omega_A|) \in [0;1]$ and $\eta_{\text{mf2}}(|\omega_A|) \in [0;1]$ denote the *mesh efficiencies* for the different power flow directions characterized by $P_A = \tau_A \,\omega_A$. Note, that the two mesh efficiencies are a function of the absolute value of ω_A. Often, $\eta_{\text{mf1}} \approx \eta_{\text{mf2}}$. However, there are also cases where the two mesh efficiences are very different, e. g., for worm gears.

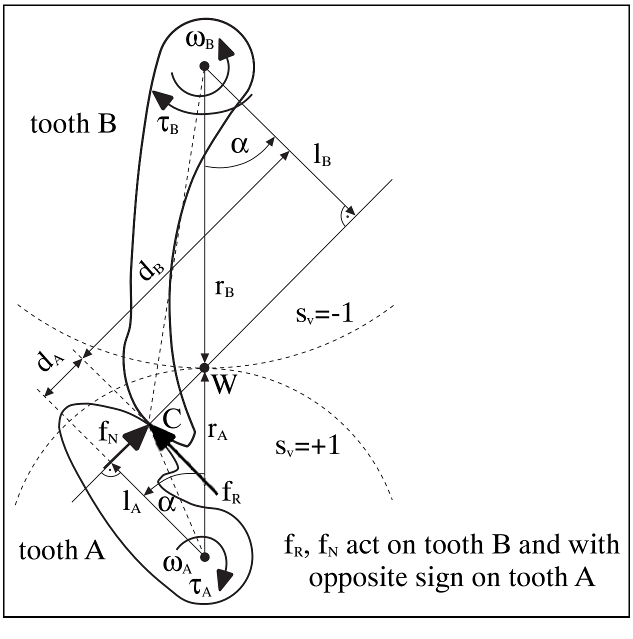

Figure 3: Friction between gear wheel teeth

2 Standard Gear

In this section, a mathematical description of the frictional effects present in a standard gear is presented and an appropriate *Modelica* model is sketched. The gear type under consideration is shown in figure 4. Here, ω_A denotes the angular velocity of the left shaft

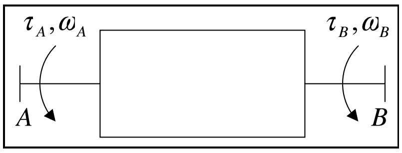

Figure 4: Speeds and cut-torques of a standard gear

and ω_B denotes the angular velocity of the right shaft, respectively. At the cut-planes of the two shafts, the constraint torques τ_A and τ_B are present. The class of gears to be examined in this section is formally defined as:

Definition 1: A gear denoted as *standard gear* in this article has the following properties:

- The gear has *two external shafts*.
- The gear has *one degree of freedom*.
- The time invariant constraint equation

$$\omega_A = i\,\omega_B \tag{6}$$

holds, where i is *constant* and not zero. This constant is called *gear ratio*[1]

This definition includes a broad class of gears.

2.1 Mathematical Description

A standard gear may have several bearings, several gear stages and several teeth in contact. Based on the observations in section 1.1 and 1.2, and assuming that all bearing losses are either transformed to bearing friction $\tau_{\text{bf,A}}$ at shaft A or bearing friction $\tau_{\text{bf,B}}$ at shaft B and that mesh friction $\hat{\eta}_{\text{mf}}$ is present, the following loss model is obtained

$$\tau_B - \tau_{\text{bf,B}} = -i\,\hat{\eta}_{\text{mf}}(\tau_A - \tau_{\text{bf,A}})\,. \tag{7}$$

Reordering of terms yields

$$\tau_B = i\,(-\hat{\eta}_{\text{mf}}\tau_A + \frac{1}{i}\tau_{\text{bf,B}} + \hat{\eta}_{\text{mf}}\tau_{\text{bf,A}})\,. \tag{8}$$

[1]The following derivation is also valid for variable gear ratios, such as a CVT gear. The only addition is that the bearing friction term $\hat{\tau}_{bf}$ is not only a function of ω_A but also of the actual gear ratio *i*, due to (8) and (9).

The torque direction of $\tau_{\text{bf,B}}$ depends on the sign of ω_B due to equation (2) and the torque direction of $\tau_{\text{bf,A}}$ depends on the sign of ω_A. Since $\omega_B = i\,\omega_A$, the torque direction of $\tau_{\text{bf,B}}/|i|$ depends also on the sign of ω_A. Therefore, all bearing friction terms can be collected to one overall bearing friction variable $\hat{\tau}_{\text{bf}}$ which has the properties that (a) the direction of this torque depends on ω_A and (b) the value depends on the same energy flow directions as $\hat{\eta}_{\text{mf}}$ does. As a result, the following gear loss model is obtained:

$$\tau_B = i\,(-\hat{\eta}_{\text{mf}}\tau_A + \hat{\tau}_{bf}) \tag{9}$$

where

$$\hat{\eta}_{\text{mf}} := \begin{cases} \eta_{\text{mf1}}(|\omega_A|) & : \begin{cases} \tau_A\,\omega_A > 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A > 0 \end{cases} \\ 1/\eta_{\text{mf2}}(|\omega_A|) & : \begin{cases} \tau_A\,\omega_A < 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A < 0 \end{cases} \\ \text{so that } \dot{\omega}_A = 0\text{:} & \omega_A = 0 \end{cases} \tag{10}$$

describes the *mesh frictions* with $\eta_{\text{mf1}}(|\omega_A|)$, $\eta_{\text{mf2}}(|\omega_A|) \in [0;1]$ and

$$\hat{\tau}_{\text{bf}} := \begin{cases} \tau_{\text{bf1}}(\omega_A) & : \begin{cases} \tau_A\,\omega_A > 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A > 0 \end{cases} \\ \tau_{\text{bf2}}(\omega_A) & : \begin{cases} \tau_A\,\omega_A < 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_A < 0 \end{cases} \\ \text{so that } \dot{\omega}_A = 0\text{:} & \omega_A = 0 \end{cases} \tag{11}$$

describes the *bearing frictions* with

$$\hat{\tau}_{\text{bf}}(\omega_A) = \begin{cases} \geq 0 & : \omega_A > 0 \\ \leq 0 & : \omega_A < 0 \end{cases}. \tag{12}$$

More detailed models are obtained by taking into account that $\eta_{\text{mf1}}(\cdot)$, $\eta_{\text{mf2}}(\cdot)$ are additionally functions of the absolute position φ_A of shaft A and of the gear temperature T, and $\hat{\tau}_{\text{bf}}(\cdot)$ is additionally also a function of T, respectively.

The model above describes especially the case when the gear bearings and the teeth in contact to each other are stuck. This occurs when $\omega = 0$. Then, $\hat{\eta}_{\text{mf}}$ and $\hat{\tau}_{\text{bf}}$ are constraint variables which are computed from the condition that the gear remains stuck, or formulated mathematically that $\dot{\omega}_A = 0$. If the constraint variables become greater as their respective sliding values at zero speed, the gear leaves the stuck mode and starts sliding. Note, that the stuck mode is both due to bearing friction (because the bearing loads introduce Coulomb friction) and due to mesh friction. Since in stuck mode there are two additional unknowns ($\hat{\eta}_{\text{mf}}$, $\hat{\tau}_{\text{bf}}$), but only one additional equation

($\dot{\omega}_A = 0$), there is an ambiguity so that either $\hat{\eta}_{\rm mf}$ or $\hat{\tau}_{\rm bf}$ can have an arbitrary value in this mode.

A direct implementation of model (9) is difficult. The key idea from [9] is to transform this model into a form close to the standard bearing friction model which is well understood. This requires to collect all loss effects in an *additive* loss torque $\Delta\tau$, i. e., to describe the mesh and bearing frictions by the equation

$$\tau_B = i\,(-\tau_A + \Delta\tau) \tag{13}$$

instead of (9). Equation (13) implicitly defines the newly introduced loss torque $\Delta\tau$, i. e., (9) and (13) are two equations for the three unknowns τ_A, τ_B and $\Delta\tau$.

In *sliding* mode, equation (9) is replaced by the combined equation of (9), (13)

$$-\hat{\eta}_{\rm mf}\,\tau_A + \hat{\tau}_{bf} = -\tau_A + \Delta\tau$$

and therefore

$$\Delta\tau = (1-\hat{\eta}_{\rm mf})\,\tau_A + \hat{\tau}_{\rm bf}\;. \tag{14}$$

In *stuck* mode, equation (9) is replaced by the constraint equation $\dot{\omega}_A = 0$.

To summarize, the transformed gear loss model is defined by (10), (11) and equations:

$$\tau_B = i\,(-\tau_A + \Delta\tau) \tag{15}$$

$$\Delta\tau = \begin{cases} (1-\hat{\eta}_{\rm mf})\,\tau_A + \hat{\tau}_{\rm bf} & : \omega_A \neq 0 \\ \text{so that } \dot{\omega}_A = 0 & : \omega_A = 0 \end{cases} \tag{16}$$

Note, that by this transformation the previous ambiguity in stuck mode is removed. Utilizing (10)-(12) in (16) for the sliding mode, results in the equations of table 1. The different regions to compute $\Delta\tau$ are visualized in the upper part of figure 5. In sliding mode, $\Delta\tau$ has either a value on the upper or on the lower limiting lines, depending on the sign of ω_A. In stuck mode, $\Delta\tau$ has a value between the limiting lines such that $\dot{\omega}_A = 0$. Stuck mode is left, when $\Delta\tau$ reaches one of the limiting lines.

ω_A	τ_A	$\Delta\tau =$
>0	≥ 0	$(1-\eta_{\rm mf1})\,\tau_A + \lvert\tau_{\rm bf1}\rvert \quad (= \Delta\tau_{\max 1} \geq 0)$
>0	<0	$(1-1/\eta_{\rm mf2})\,\tau_A + \lvert\tau_{\rm bf2}\rvert \quad (= \Delta\tau_{\max 2} \geq 0)$
<0	≥ 0	$(1-1/\eta_{\rm mf2})\,\tau_A - \lvert\tau_{\rm bf2}\rvert \quad (= \Delta\tau_{\min 1} \leq 0)$
<0	<0	$(1-\eta_{\rm mf1})\,\tau_A - \lvert\tau_{\rm bf1}\rvert \quad (= \Delta\tau_{\min 2} \leq 0)$

Table 1: $\Delta\tau = \Delta\tau(\omega_A, \tau_A)$ in sliding mode

In the lower part of figure 5 the torque loss $\Delta\tau$ is shown using ω_A as abscissa and τ_A as curve parameter.

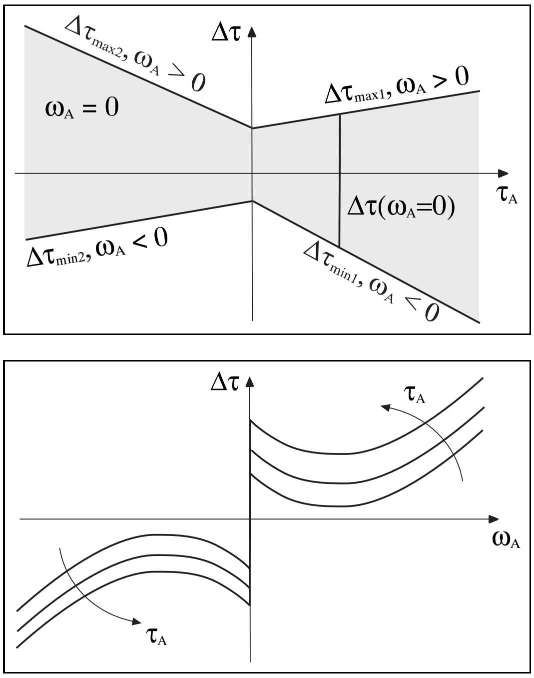

Figure 5: $\Delta\tau$ in sliding and stuck mode

By this figure it can be clearly seen, that the transformation to $\Delta\tau$ results in a friction characteristic which is close to a pure bearing friction model. The stuck mode is described in an identical way. Only the sliding friction torque is no longer a function of solely ω_A, but additionally a function of τ_A.

2.2 Modelica Model

The gear loss model derived in the previous section can be implemented as a *Modelica* model in a straightforward manner. The parameters to be provided are the gear ratio `i` and table `lossTable` to define the gear losses, see table 2. Tabulated values of the variables $\eta_{\rm mf1}$, $\eta_{\rm mf2}$, $\tau_{\rm bf1}$, $\tau_{\rm bf2}$ have to be given as function of $\omega_A \geq 0$. The values for negative ω_A are automatically taken care off. Whenever $\eta_{\rm mf1}$, $\eta_{\rm mf2}$, $\hat{\tau}_{\rm bf1}$ or $\hat{\tau}_{\rm bf2}$ are needed, they are determined by interpolation in `lossTable`. The interface of this *Modelica* model is therefore defined as

```
parameter Real i = 1;
parameter Real lossTable[:,5]
                = [0, 1, 1, 0, 0];
```

$\lvert\omega_A\rvert$	$\eta_{\rm mf1}$	$\eta_{\rm mf2}$	$\lvert\tau_{\rm bf1}\rvert$	$\lvert\tau_{\rm bf2}\rvert$
⋮	⋮	⋮	⋮	⋮

Table 2: Format of table `lossTable`.

using the unit gear ratio and no losses as a default.

Internally, the gear loss model is based on the `BearingFriction` model, because this model already implements the sliding/stuck handling in a satisfactory way. The only enhancement is that the sliding friction torque $\Delta\tau$ is not only a function of ω_A but also of the unknown variable τ_A and of the relations $\tau_A \geq 0$, $\tau_A < 0$.

During code generation, this results in an additional algebraic loop in which τ_A, $\Delta\tau$ and the two relations are contained. Due to the structure of equation (14), the continuous unknowns (τ_A, $\Delta\tau$) enter this loop linearly. The resulting algebraic loop is a mixed Real/Boolean system of equations, which is very similiar to the corresponding mixed system of equation of a pure bearing friction model, and can be solved with the same methods, see, e. g., [8].

2.3 Measurement of Gear Losses

Efficiency measurement data provided in gearbox catalogues contain usually not enough information for a dynamic simulation (e. g., the losses for $\omega_A = 0$ are not given). The reason is that in many cases only the overall efficiency is included as a function of load torque for some constant angular velocities $\omega_A \neq 0$.

In order to obtain the data needed for the `lossTable` of the *Modelica* model, the following measurement method is proposed:

> For $m \geq 2$ fixed load torques $\tau_{B,j}$ (e. g., nominal torque and half of the nominal torque) and n angular velocities $\omega_{A,k}$, the necessary driving torques τ_A are measured which are needed to drive the gear for positive and negative energy flow $P_A = \omega_A \tau_A$, including measurements near $\omega_A = 0$ (= the gear shafts start to rotate).

As a result, the following values are obtained:

$$\tau_{A,j}(\omega_{A,k}) , \quad \tau_{B,j} \qquad (j = 2..m , \quad k = 1..n) .$$

For every fixed speed $\omega_{A,k}$, equation (9) can be formulated in the unknowns $\hat{\eta}_{\text{mf}}$ and $\hat{\tau}_{\text{bf}}$. Collecting all equations together results in one linear system of equations

$$\begin{bmatrix} -i\tau_{A,1}(\omega_{A,k}) & 1 \\ -i\tau_{A,2}(\omega_{A,k}) & 1 \\ \vdots & \vdots \\ -i\tau_{A,m}(\omega_{A,k}) & 1 \end{bmatrix} \begin{bmatrix} \hat{\eta}_{\text{mf}} \\ \hat{\tau}_{\text{bf}} \end{bmatrix} = \begin{bmatrix} \tau_{B,1} \\ \tau_{B,2} \\ \vdots \\ \tau_{B,m} \end{bmatrix} \tag{17}$$

for every fixed speed $\omega_{A,k}$. If more than two load torque measurements are available, (17) has no solution and is solved in a least square sense. For two load torque measurements, a unique solution exists

$$\hat{\eta}_{\text{mf}}(\omega_{A,k}) = -\frac{\tau_{B,1} - \tau_{B,2}}{i(\tau_{A,1} - \tau_{A,2})} \tag{18}$$

$$\hat{\tau}_{\text{bf}}(\omega_{A,k}) = \frac{1}{i}\tau_{B,2} + \hat{\eta}_{\text{mf}}\tau_{A,2} . \tag{19}$$

Finally, $\eta_{\text{mf1}}(\omega_{A,k})$, $\eta_{\text{mf2}}(\omega_{A,k})$, $\tau_{\text{bf1}}(\omega_{A,k})$ and $\tau_{\text{bf2}}(\omega_{A,k})$ can be easily determined from $\hat{\eta}_{\text{mf}}(\omega_{A,k})$ and $\hat{\tau}_{\text{bf}}(\omega_{A,k})$ based on the sign of $\omega_A \tau_A$ using equations (10) and (11).

As already mentioned, in gearbox catalogues usually the overall efficiency $\eta = -P_B/P_A$ is provided as function of the load torque τ_B. To demonstrate that the presented loss model produces qualitatively the same result, the overall efficiency of the following example with the loss model

$$\eta_{\text{mf1}} = 0.97$$

$$\tau_{\text{bf1}}/\tau_{Bmax} = 0.01\,(\omega_B^2 + 2\,\omega_B + 5)$$

is shown in figure 6. As can be seen, the typical hyperbolical curves are present, although the mesh efficiency is constant.

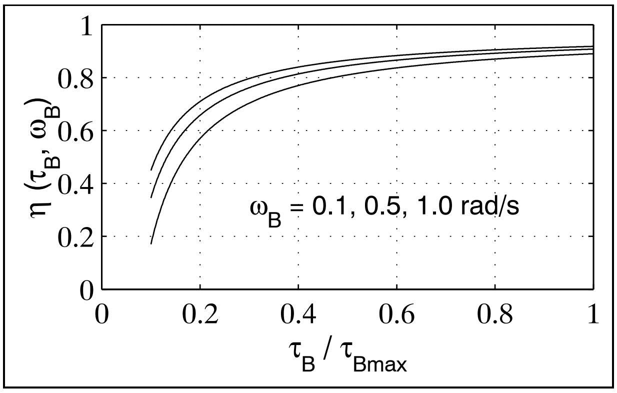

Figure 6: Overall efficiency as function of τ_B.

3 Planetary Gear

In this section the frictional effects of planetary gears are mathematically described in a similiar way as in

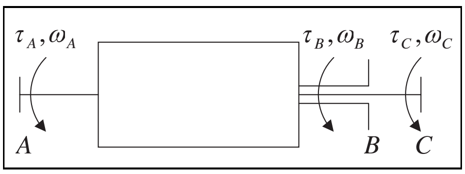

Figure 7: Speeds and cut-torques of a planetary gear

the previous section (based on [9]) and an appropriate *Modelica* model is derived. The variables describing a planetary gear are shown in figure 7, where ω_A, ω_B, ω_C denote the angular velocities of shafts A, B, C and τ_A, τ_B, τ_C denote the torques at the cut planes of the shafts, respectively. The examined gears are defined as follows:

> *Definition 2:* A gear denoted as *planetary gear* in this article has the following properties:
>
> - The gear has *three external shafts*.
> - The gear has *two degrees of freedom*.
> - The time invariant constraint equation
>
> $$\omega_{AB} = i_0\, \omega_{CB} \qquad (20)$$
>
> holds, with
>
> $$\omega_{AB} = \omega_A - \omega_B$$
> $$\omega_{CB} = \omega_C - \omega_B$$
>
> where the so-called stationary gear ratio i_0 is *constant* and is in the range $i_0 \leq -1$ or $i_0 > 1$ (for i_0 outside of this range, the role of shafts A and C has just to be exchanged. However, $i_0 = 0$ and $i_0 = 1$ is never possible).

Note, that Willis' equation, see, e. g. [4], is equivalent to (20). A large class of planetary gears matches to this definition. Some examples are shown in figure 8. The stationary gear ratio i_0 is usually computed from the teeth number of the gear wheels. For example, for the gearbox in the left upper corner of figure 8, $i_0 = z_r/z_s$, where z_s is the number of teeth for the inner sun wheel and z_r is the number of teeth for the outer ring wheel (teeth numbers are taken negative for internal teeth).

3.1 Mathematical Description

The relationship between the angular velocities of the three shafts shown in figure 7 can be described using relative kinematics yielding

$$\omega_A = \omega_{B0} + \omega_{AB} \qquad (21)$$
$$\omega_B = \omega_{B0} \qquad (22)$$
$$\omega_C = \omega_{B0} + \omega_{CB}\ . \qquad (23)$$

The structure of (21)-(23) reflects the superposition of two movement types:

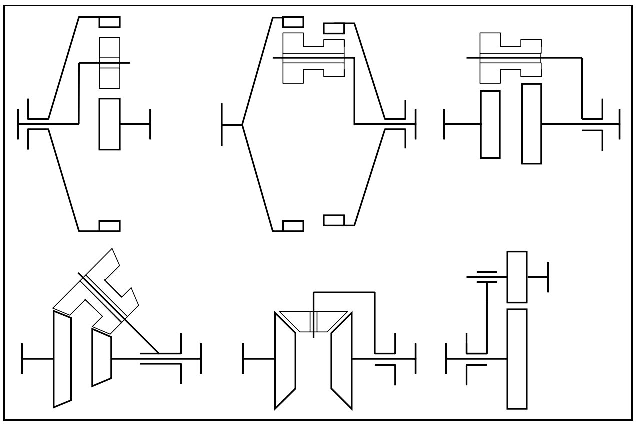

Figure 8: Examples of planetary gears according to definition 2 [4]

- *Block movement.* The whole gear rotates as one fixed block with angular velocity ω_{B0}:

  $$\omega_A = \omega_B = \omega_C = \omega_{B0}\ .$$

  During this movement a power P_1 is transmitted solely by rigid coupling of the three shafts. As the three shafts are not rotating relative to each other, any losses due to friction between the teeth of the gear wheels or in internal bearings cannot arise, i. e., the block movement is without losses.

- *Stationary gear movement.* Shaft B is fixed relative to the inertial system:

  $$\omega_{B0} = 0 \Rightarrow \omega_A = \omega_{AB} \quad \omega_C = \omega_{CB}\ .$$

  During this movement a power P_2 is transmitted solely by sliding of teeth in all three gear wheels resulting in power losses due to friction between the teeth of the gear wheels and in internal bearings not related to the external shafts. Since the shaft speeds are a function of ω_{AB}, ω_{CB} and $\omega_{CB} = \omega_{AB}/i_0$ due to (20), losses only occur, if $\omega_{AB} \neq 0$.

In order to achieve further equations energy flow conservation is considered according to figure 9 involving

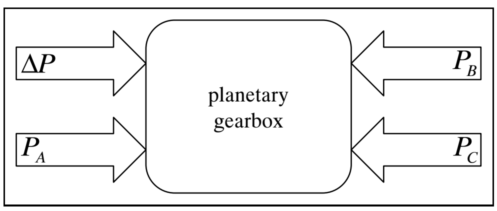

Figure 9: Energy flow

the energy flows in the shafts, P_A, P_B, P_C, and the friction losses ΔP which dissipate to heat:

$$P = P_A + P_B + P_C + \Delta P = 0 \tag{24}$$

with

$$P_A = \tau_A\,(\omega_{B0} + \omega_{AB}) \tag{25}$$
$$P_B = \tau_B\,\omega_{B0} \tag{26}$$
$$P_C = \tau_C\,(\omega_{B0} + \omega_{CB})\;. \tag{27}$$
$$\Delta P = -\Delta\tau\,\omega_{AB}\;. \tag{28}$$

Since losses can only occur if $\omega_{AB} \neq 0$, the power loss ΔP has been formulated as the product of ω_{AB} and a, yet unknown, virtual loss torque $-\Delta\tau$. Using (24)-(27) and (20) results in

$$P = \underbrace{\omega_{B0}\,(\tau_A + \tau_B + \tau_C)}_{P_1} + \underbrace{\omega_{AB}\,(\tau_A + \tau_C/i_0 - \Delta\tau)}_{P_2} \tag{29}$$

Since a planetary gearbox has two degrees of freedom (see definition 2), the two speeds ω_{B0} and ω_{AB} can have arbitrary values which are independent from each other. Therefore the speed factors must vanish

$$0 = \tau_A + \tau_B + \tau_C \tag{30}$$
$$0 = \tau_A + \tau_C/i_0 - \Delta\tau \tag{31}$$

By solving (31) for τ_C and (30) for τ_B, the two equations can be alternatively formulated as

$$\tau_C = i_0(-\tau_A + \Delta\tau) \tag{32}$$
$$\tau_B = (i_0 - 1)\,\tau_A - i_0\,\Delta\tau \tag{33}$$

When stationary gear movement occurs, i.e., $\omega_{B0} = 0$, the planetary gear reduces to a standard gear with two external shafts where the losses are described according to equation (9)

$$\tau_C = i_0\,(-\hat{\eta}_{\text{mf}}\,\tau_A + \hat{\tau}_{\text{bf}}) \tag{34}$$

where $\hat{\eta}_{\text{mf}}(\omega_{AB})$ describes mesh friction

$$\hat{\eta}_{\text{mf}} := \begin{cases} \eta_{\text{mf1}}(|\omega_{AB}|) & : \begin{cases} \tau_A\,\omega_{AB} > 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_{AB} > 0 \end{cases} \\ 1/\eta_{\text{mf2}}(|\omega_{AB}|) & : \begin{cases} \tau_A\,\omega_{AB} < 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_{AB} < 0 \end{cases} \\ \text{so that } \dot{\omega}_{AB} = 0\text{:} & \omega_{AB} = 0 \end{cases} \tag{35}$$

with $\eta_{\text{mf1}}, \eta_{\text{mf2}} \in [0;1]$ and $\hat{\tau}_{\text{bf}}$ describes friction in the internal bearings of the planetary gearbox (e. g., for a standard planetary gearbox with sun, planet and ring wheel, $\hat{\tau}_{\text{bf}}(\omega_{AB})$ is the bearing friction torque in the planet bearings)

$$\hat{\tau}_{\text{bf}} := \begin{cases} \tau_{\text{bf1}}(\omega_{AB}) & : \begin{cases} \tau_A\,\omega_{AB} > 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_{AB} > 0 \end{cases} \\ \tau_{\text{bf2}}(\omega_{AB}) & : \begin{cases} \tau_A\,\omega_{AB} < 0 \textbf{ or} \\ \tau_A = 0 \textbf{ and } \omega_{AB} < 0 \end{cases} \\ \text{so that } \dot{\omega}_{AB} = 0\text{:} & \omega_{AB} = 0 \end{cases} \tag{36}$$

with

$$\hat{\tau}_{\text{bf}}(\omega_{AB}) = \begin{cases} \geq 0 & : \omega_{AB} > 0 \\ \leq 0 & : \omega_{AB} < 0 \end{cases}\;. \tag{37}$$

No losses will be additionally introduced when a block movement is superpositioned, as discussed previously. Therefore, (34) is also valid for a general movement. Comparision of (34) with (32) results in

$$i_0(-\tau_A + \Delta\tau) = i_0\,(-\hat{\eta}_{\text{mf}}\,\tau_A + \hat{\tau}_{\text{bf}})$$

and therefore

$$\Delta\tau = (1 - \hat{\eta}_{\text{mf}})\,\tau_A + \hat{\tau}_{\text{bf}}\;. \tag{38}$$

As energy can dissipate only,

$$\Delta P = -\omega_{AB}\,\Delta\tau \overset{!}{\leq} 0 \tag{39}$$
$$= -\omega_{AB}\,((1 - \hat{\eta}_{\text{mf}})\,\tau_A + \hat{\tau}_{\text{bf}}) \tag{40}$$
$$= -(1 - \hat{\eta}_{\text{mf}})\,\tau_A\,\omega_{AB} - \omega_{AB}\,\hat{\tau}_{\text{bf}} \tag{41}$$

Since

$$1 - \hat{\eta}_{\text{mf}} \geq 0 \quad \text{for} \quad \tau_A\,\omega_{AB} \geq 0$$
$$1 - \hat{\eta}_{\text{mf}} \leq 0 \quad \text{for} \quad \tau_A\,\omega_{AB} \leq 0$$

according to (35), the first term in (41) is never positive. With (37) the same also holds for the second term and therefore ΔP is in fact never positive. Utilizing (35)-(37) and (38) for the sliding case, results in the equations of table 3 to actually calculate $\Delta\tau$.

In the stuck mode the planetary gear rotates without any losses as a block. Similar to Sec. 2 the torque loss $\Delta\tau$ is then defined implicitly by the constraint equation $\dot{\omega}_{AB} = 0$. The gear remains in sliding mode until ω_{AB} becomes zero. It remains in stuck mode as long as the calculated torque loss $\Delta\tau$ is lying in the stuck region according to figure 10.

ω_{AB}	τ_A	$\Delta\tau =$
> 0	≥ 0	$(1 - \eta_{\text{mf}})\,\tau_A + \lvert\tau_{\text{bf1}}\rvert \;\; (= \Delta\tau_{\max 1} \geq 0)$
> 0	< 0	$(1 - 1/\eta_{\text{mf}})\,\tau_A + \lvert\tau_{\text{bf2}}\rvert \;\; (= \Delta\tau_{\max 2} \geq 0)$
< 0	≥ 0	$(1 - 1/\eta_{\text{mf}})\,\tau_A - \lvert\tau_{\text{bf2}}\rvert \;\; (= \Delta\tau_{\min 1} \leq 0)$
< 0	< 0	$(1 - \eta_{\text{mf}})\,\tau_A - \lvert\tau_{\text{bf1}}\rvert \;\; (= \Delta\tau_{\min 2} \leq 0)$

Table 3: $\Delta\tau = \Delta\tau(\omega_{AB}, \tau_A)$ in sliding mode

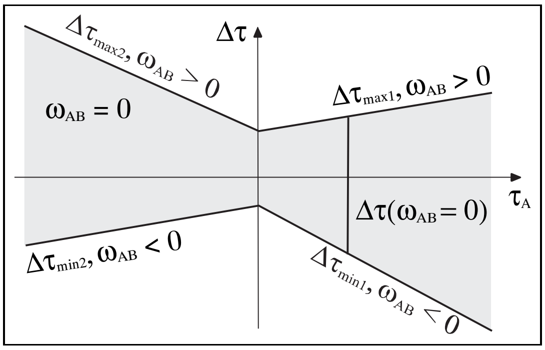

Figure 10: $\Delta\tau$ in sliding and stuck mode

$\lvert\omega_{AB}\rvert$	η_{mf1}	η_{mf2}	$\lvert\tau_{bf1}\rvert$	$\lvert\tau_{bf2}\rvert$
⋮	⋮	⋮	⋮	⋮

Table 4: Format of table `lossTable`

3.2 Modelica Model

The planetary gear loss model derived in this section can be implemented as a *Modelica* model in a similar way as described in Sec. 2. The parameters to be provided are the stationary gear ratio i_0 and table, `lossTable` to define the gear losses, see table 4.

Whenever η_{mf1}, η_{mf1}, τ_{bf1} or τ_{bf2} are needed, they are determined by interpolation in `lossTable`. The interface of this *Modelica* model is therefore defined as

```
parameter Real i = 1;
parameter Real lossTable[:,5]
                = [0, 1, 1, 0, 0];
```

using the unit gear ratio and no losses as a default. The comments about model interna given in Sec. 2 are valid similarily for the planetary gear model.

This Modelica model can be connected with component `Modelica.Mechanics.Rotational.-BearingFriction` at each shaft to model additionally the friction influence of bearings related to the external shafts. As a consequence multiple friction phases arise with the phenomena explained, e. g., in [8].

4 Simulation Results

In this section simulation results are presented for models containing the standard and planetary gear models with losses developed in the last sections, using the Modelica modeling and simulation environment Dymola, version 4.2a [1].

4.1 Standard gear with mesh friction

Figure 11 contains a Dymola screenshot of the model under consideration. It is a standard gear with mesh friction that is driven by a sinusoidal torque and has a load torque which is linearly increasing.

In figure 12 and 13 results of a simulation are shown for $\eta_{mf} = 0.5$ and $\eta_{mf} = 0.9$: The thicker lines are the loss torques $\Delta\tau$ whereas the thinner lines characterize whether mesh friction is in forward sliding (`mode=+1`), backward sliding (`mode=-1`) or stuck (`mode=0`) mode.

As can be seen in the upper part of figure 12 from the two stuck modes, the maximum loss torque is not constant (as it is for bearing friction) but depends on the driving torque. Additionally, figure 13 contains the speed of inertia 2 for $\eta_{mf} = 0.5$. During stuck mode, the velocity vanishes.

4.2 Gear shift dynamics of automatic gear

The mesh friction model for planetary gears as well as the clutch friction model already available in the Mod-

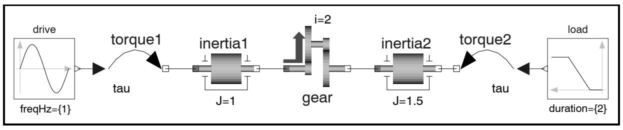

Figure 11: Modelica composition diagram of gear with mesh friction.

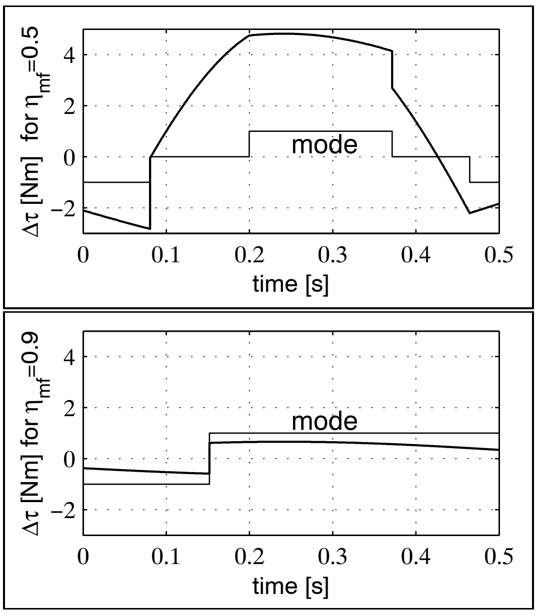

Figure 12: Loss torque $\Delta\tau$ and `mode` in gear with mesh friction $\eta_{mf} = 0.5$ and $\eta_{mf} = 0.9$.

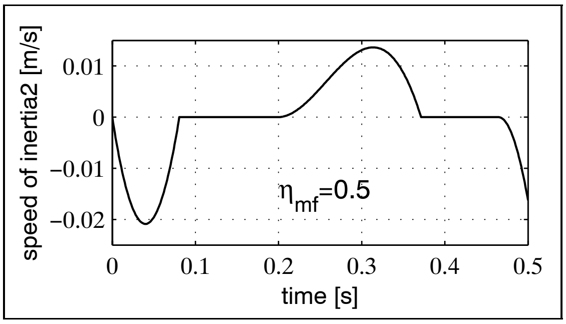

Figure 13: Speed of inertia 2 for $\eta_{mf} = 0.5$.

elica standard library are very well suited to simulate the shift dynamics of automatic gearboxes reliably and efficiently. As an example, the shift dynamics of the automatic gearbox ZF 4HP22 is examined in more detail. A schematic together with the gear shift table is given in figure 14 (from [2]).

From this schematic it is straightforward to build up the Modelica composition diagram from figure 18 containing the clutches, combined clutches and free wheels, and the three planetary gears with mesh friction ($\eta_{mf} = 0.975$).

In order that simulations can be performed, a model of the environment in which the automatic gear operates is needed. A typical example of a longitudinal dynamics model of a vehicle is shown in figure 19. It consists of a driver, an engine, an automatic gearbox with transmission control unit, an axle and a simple 1-dimensional vehicle model containing the most important drive resistances.

Signals, such as desired vehicle velocity or throttle position, are transported between the components by a signal bus. Via `send` and `receive` blocks, signals can be send to or received from the bus connector `bus`

Gear	C4	C5	C6	C7	C8	C11	C12
1	x					x	
2	x		x	x		x	
3	x	x		x		x	
4	x	x		x			x
R		x			x	x	

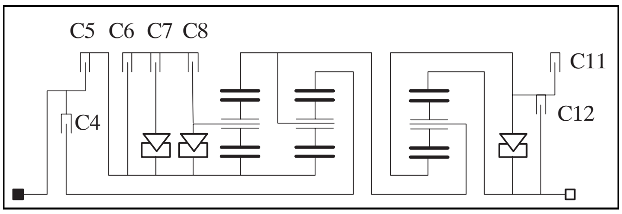

Figure 14: Gear shift table for gearbox ZF 4HP22.

which is a Modelica connector containing declarations of all variables present in the bus.

Typical simulation results of the model are shown in figure 15-17, especially in figure 15 the desired and actual velocity of the vehicle in km/h, in figure 16 the vehicle acceleration and the actual gear determined by the simple transmission control unit, and in figure 17 the torque loss $\Delta\tau$ of the right most planetary gearbox `p3`.

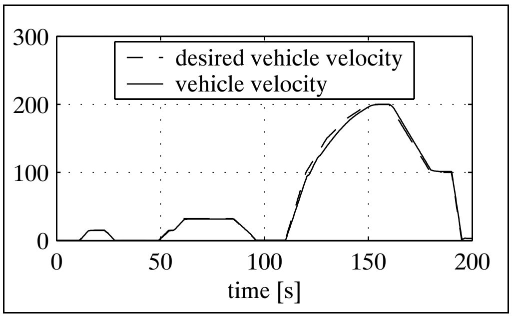

Figure 15: Desired and actual velocity of vehicle.

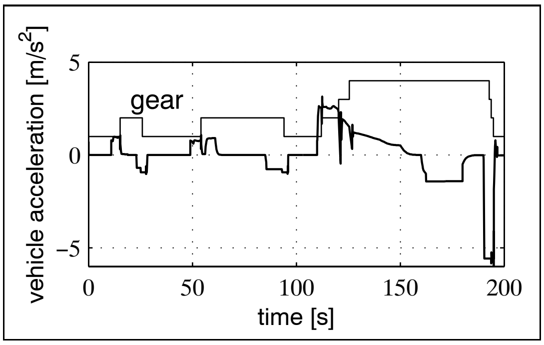

Figure 16: Vehicle acceleration and actual gear.

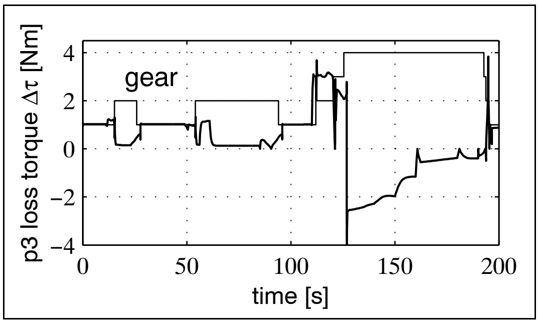

Figure 17: Loss torque $\Delta\tau$ in planetary gear p3.

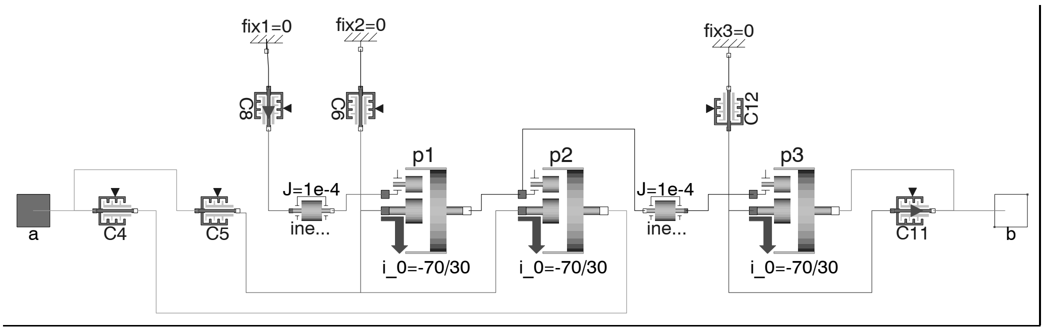

Figure 18: Modelica composition diagram of automatic gearbox ZF 4HP22.

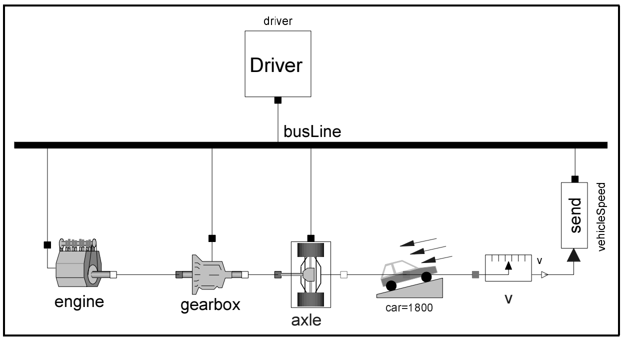

Figure 19: Modelica composition diagram of vehicle longitudinal dynamics.

5 Conclusions and Outlook

A loss model for a broad class of standard gears and planetary gears has been presented which includes Coulomb friction in the gearbox bearings and Coulomb friction between the gear teeth. Most important, the locking and unlocking of the friction elements are handeled, including the friction between the gear teeth. This allows to model and simulate the stick-slip effect of standard and planetary gears as function of the shaft speeds and the driving or load torque which is essential for the design of servo drives.

The usual approach to model mesh friction as an element which switches between two different efficiencies, leads in such situations to *chattering*, i. e., very fast switching between the two possible modes which in turn results in very small step sizes and practically stops the simulation. The new approach described here will lead to much more reliable and more efficient simulations.

The gear losses in standard gears have been implemented in a new model `LossyGear` which will be available in the next version of the `Modelica.Mechanics.Rotational` library. The gear loss model for planetary gears has been implemented in a new model `LossyPlanetary` which will be available in the next version of the `PowerTrain` library.

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