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Simulating permanent magnet brushless motors in DYMOLA

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Abstract

Multi-domain dynamic simulation is becoming an issue in the design of high performance mechatronic systems, where advances are foreseen only if the mutual interaction of different parts of the system is well understood. The modelling environment provided by DYMOLA with Modelica language proved to be ideal for studying the mutual effects of mechanics, electronics and control in a brushless motor, whose model has been conceived as one of the building blocks of a wider project, aimed at simulating a complete machining centre. Details on the model of the brushless motor as well as on its simulation are given in the present paper.

1 Introduction

The most common actuation systems adopted in robotics, machine tools industry and machining centers are by far servomechanisms with permanent magnet brushless motors, connected to the loads by transmission chains (or gearboxes).

In a brushless motor the electromechanical commutation typical of brushed DC motors is replaced by an electronic commutation of the currents in the three phases of the stator windings. This should in principle guarantee that the electromagnetical torque delivered on the motor shaft is independent of the rotor position. However some constructive imperfections in the motor or in the drive, where electronic commutation is implemented, produce an undulation (ripple) [4] on the actual torque. While this problem could be considered minor in the static dimensioning of the actuation system, it is of utmost importance for its dynamic performance. Torque ripple might in fact excite the resonances of the mechanical system, usually associated to the elastic couplings between motors and loads. Dynamic simulation [2], or virtual prototyping in a more recent jargon, is a valuable tool to study these phenomena, and in particular to separate the effects of the single sources of disturbances on the performance of the system. Mechanics, electronics and control are different domains involved in this truly mechatronic problem. *Multi-domain simulation environments* are required to simulate with a reasonable effort the system, while the particular electrical configuration of the stator windings (Y connected) calls for the adoption of modelling languages where *algebraic constraints* on state variables can be easily specified.

DYMOLA (with Modelica language [7, 5]) has been found to fit easily both the above requirements. Mechanical, electrical and control systems can be combined in a natural and physics-driven way, while the acausal modelling based on DAE equations, proper of this environment, allows to specify the constraint on the phase currents as it is, avoiding reformulation of the system's equations in terms of two out of three currents, typical of procedural modelling languages.

In the present work DYMOLA has been used to simulate a brushless motor controlled with an analogue driver and with a full digital driver. The simplified model ([3]) of torque ripple has been validated through these simulations. The model of the brushless motor with its analogue or digital drivers has been actually used as one of the building blocks of a wider project, where the simulation of a complete machining center (detailed simulation of the mechanical parts of the system and of various features of the CN) has been implemented.

2 Torque ripple modelling

The functional scheme of a sinusoidal PMAC machine is represented in Fig. 1. If a reference torque $\bar{\tau}$ should be delivered by the motor, typically as re-

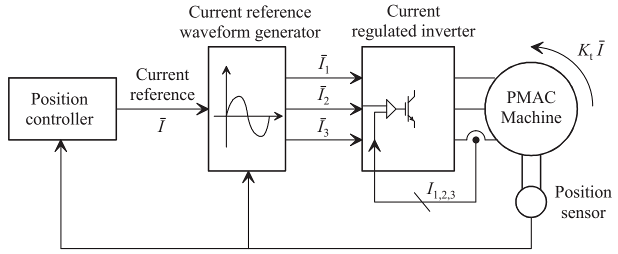

Figure 1: Functional scheme of a brushless motor

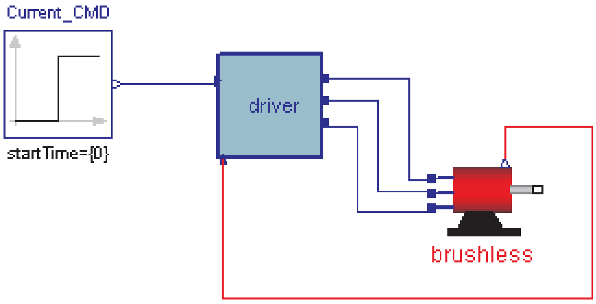

Figure 2: Complete model of the system

quired by a position controller, the current reference $\bar{I}$ has to be given the value $\bar{I} = \bar{\tau}/K_t$, where K_t is the torque constant. This scalar setpoint is then modulated through three sinusoidal functions of the electrical angle $\alpha = pq_m$, p being the number of pole pairs and q_m being the motor angle, that are offset by an angle $2\pi/3$ one from each other. The three resulting signals become the current references for the three phases. High bandwidth current controllers make the currents track their setpoints in each phase (actually two out of the three Y connected phases are closed loop controlled). If the current reference in each phase is given the same dependence on the electrical angle characterizing the back EMF (ideally sinusoidal or trapezoidal), a torque τ is produced, approximately equal (in a band of frequencies limited by the current loops) to the desired torque $\bar{\tau}$, and thus proportional to the scalar current reference $\bar{I}$.

Brushless motors, however, introduce a disturbance in the system in the form of a ripple on the torque. Several constructive imperfections of the motor and the servodrive sum up to form this pulsating disturbance. Examples are cogging torque, offsets in the current sensors, imperfections in the construction of the motor and the drive, implying that both the back EMF profiles and the phase currents may be affected by undesired higher order harmonics.

As it is shown in [3], the following relation can be used to represent in a compact form the effects of the disturbances on the torque production:

$$\tau = \tau(\alpha, \bar{I}) = \gamma(\alpha) + K_t\bar{I}(1 + \delta(\alpha)) \qquad (1)$$

The term $\gamma(\alpha)$ accounts for the disturbances due to the cogging torque and to the current offset in the drives, while the second term is responsible for the nominal torque (with $\delta(\alpha) = 0$) and for the disturbances related to the harmonic content. It is also possible to include in $\delta(\alpha)$ the effects of the amplitude imbalances and the phase misalignments of the current and back EMF shapes profiles [3].

3 Modelling the system in DYMOLA

The model of the system is obtained by the feedback connection of two sub-models, one representing the brushless motor, the other one the driver (Fig. 2). The two models are connected through three electrical connectors (the three phases of the motor) as well as through a control connector (the measure of the rotor position).

The brushless model is shown in Fig. 3. The three phases are Y connected in the block emf3, that generalizes the EMF model in the Modelica.Electrical.Analog.Basic library. In the emf3 model the back-emf profiles on the single phases are assigned. The nominal sinusoidal profiles can then be modified to study ripple due to higher order harmonics. The torque at the flange of the emf3 model derives from the equilibrium with the sum of the products of currents and back emf profiles on the single phases. Remarkably, the acausal modelling environment provided by DYMOLA allows to specify in the most natural way the algebraic constraint on the currents (the sum of the currents must be zero). This constraint would obviously generate troubles in other simulation environments based on causal specifications of the models, expressed with ODE systems. Just for comparison, Fig. 4 shows the SIMULINK model of the electrical part of a brushless motor, obtained by resolving the algebraic constraint and expressing the whole model in terms of two out of three currents. The derivation of the model is time consuming and error prone and the result lacks readability.

Modelling of the mechanical part of the motor is on the other hand standard.

The analogue version of the current controller is shown in Fig. 5. The current reference (usually the output of a position/velocity controller) is modulated through sinusoidal functions of the electrical angle, to form the references for two of the three Y connected currents. Current sensors are included in the drive and

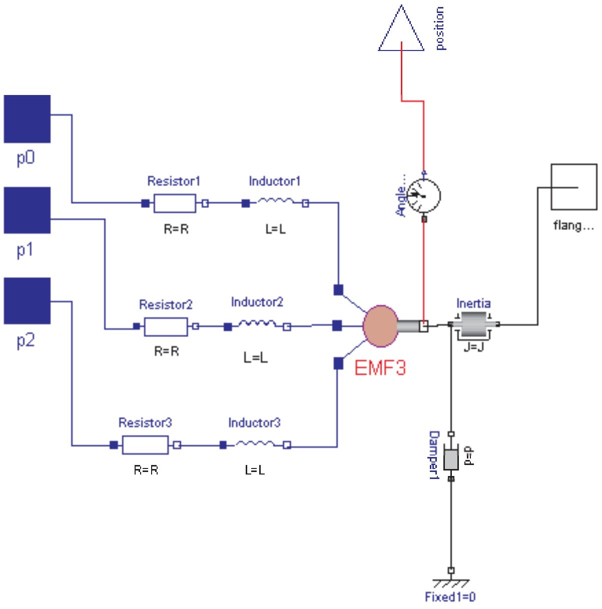

Figure 3: Model of the brushless motor

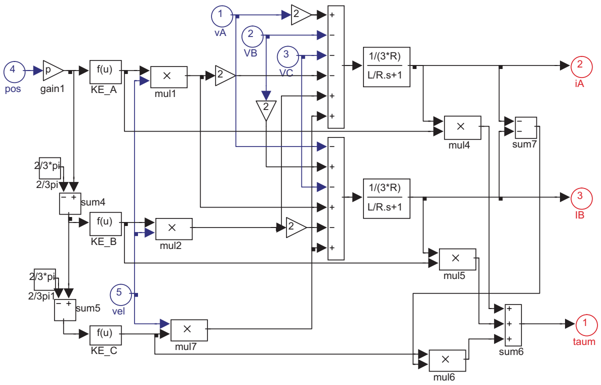

Figure 4: Model of the brushless motor in SIMULINK

possible offsets can be added to the measures, in order to simulate their effects on the generation of torque ripple. Two anti-windup PI controllers close the current loops. Their outputs are then linearly amplified and form the voltages to be applied to the single phases of the motor.

A full digital version of the current controller has been implemented as well (Fig. 6). A vector control scheme [6] has been adopted, where the phase currents are first transformed into direct and quadrature currents through Park's transformation. Two digital loops are closed on these currents, the quadrature reference being the output of the outer position/velocity controller, the direct reference being zero as usual. The voltage commands output of the two antiwindup PI controllers are then back transformed to voltages on the three phases through inverse Park's transformation.

The PWM amplifier has not been simulated since, operating with a frequency 10 or $20kHz$ and with a modulation of the pulse width of $1\mu s$, it requires an integration step size less than $1\mu s$, which might be acceptable for the simulation of the electronics of a motor drive but is far too small in the combined simulation of the mechanics and the electronics. This is particularly true if the model of the motor is instantiated several times, for the simulation of a complete machine.

4 Simulating the motor without load

Simulations obtained with the analogue version of the driver will be presented here, in order to show the utility of the model. The input of the system is a step on the current reference: the signal is expressed in Volt and has been given the value $1V$ (corresponding to 10% of the entire scale and to a current of $1.9A$). The current-to-torque gain of the motor (K_t) is equal to $1.1Nm/A$, while the number of pole pairs is equal to 3. Both the current loops have been tuned for a bandwidth of $1kHz$. The inertia of the motor is equal to $0.012Kgm^2$ while the damping factor is $0.371Nms/rad$.

Fig. 7 and Fig. 8 show, on different time scales, the responses of the electromagnetic torque and of the motor velocity in nominal conditions. The responses match the expectations [1] both from the transient point of view and from the steady state one (no oscillations is produced).

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[1] The negative sign of the torque is of no particular meaning, being associated just to the way the balance of torques is written in the block emf3.

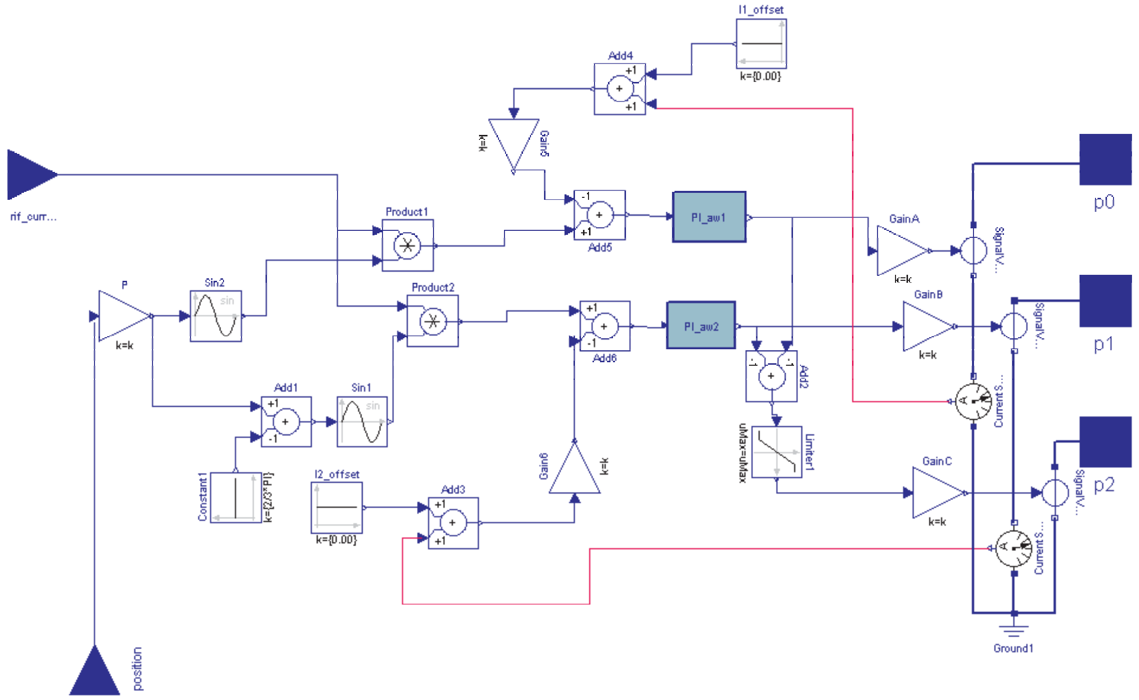

Figure 5: Model of the analogue driver

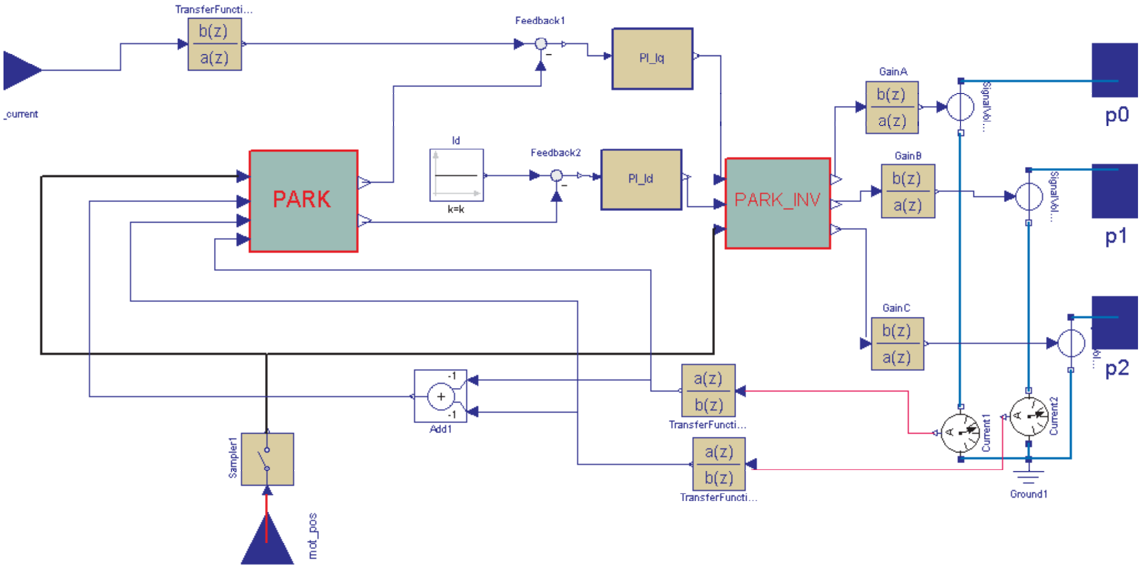

Figure 6: Model of the digital driver

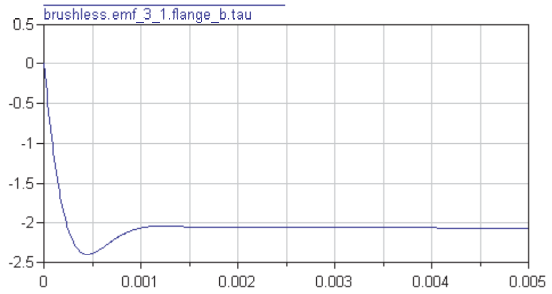

Figure 7: Electromagnetic torque in nominal conditions

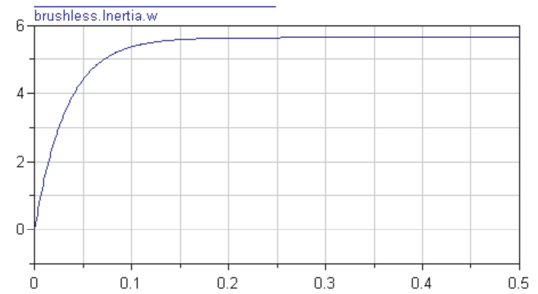

Figure 8: Motor velocity in nominal conditions

In a second simulation, an offset of 1% of the rated current has been introduced on both the current sensors. The resulting electromagnetic torque is shown in Fig. 9. As the average velocity is equal to the value obtained in nominal conditions $\Omega \approx 5.6 rad/s$, the periodicity of the disturbance is consistent with theory [3] ($T = 2\pi/(3\Omega) = 0.37 rad/s$).

The effect of higher order harmonics in the back e.m.f. profiles has been simulated introducing the fifth harmonic on all the three profiles, with amplitude 5% of the main harmonic and no misalignments or unbalances. The result in terms of electromagnetic torque is reported in Fig. 10. Again the periodicity of the disturbance is consistent with theory [3] ($T = 2\pi/(18\Omega) = 0.062 rad/s$).

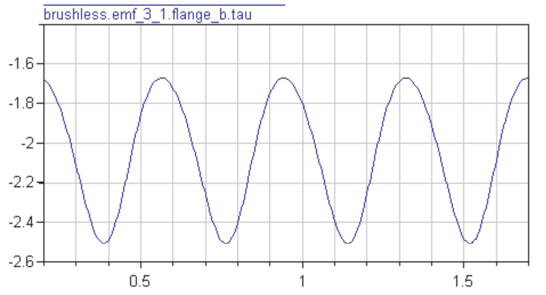

Figure 9: Electromagnetic torque with a current offset

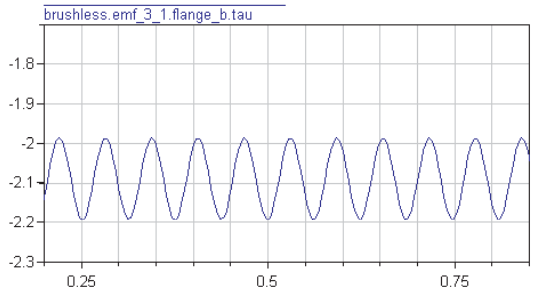

Figure 10: Electromagnetic torque with a high order back emf harmonic

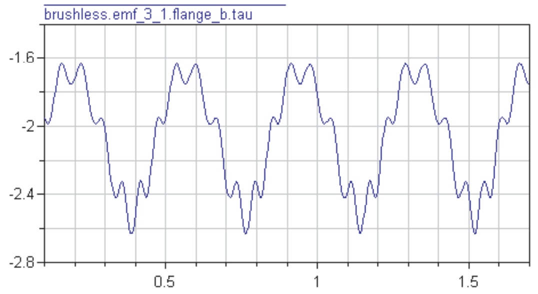

Figure 11: Electromagnetic torque with both disturbances

The superimposition of the two disturbances (the offset on the current sensor and the high order back emf harmonic) yields the electromagnetic torque reported in Fig. 11.

It is not difficult to verify (for example exporting the results of the simulation in Matlab) that the above torque profile corresponds, apart from the sign inversion, to (1), where:

$$\gamma(\alpha) = 3KI_{off}\sin(\alpha + \frac{2}{3}\pi) \tag{2}$$

$$\delta(\alpha) = -\frac{K_5}{K}\cos(6\alpha) \tag{3}$$

where I_{off} is the current offset, K_5 is the amplitude of the fifth harmonic of the back emf profile, K is the amplitude of the main harmonic ($K = 2/3K_t$).

5 Simulating the motor with a load

As already mentioned in the Introduction, one of the reasons why torque ripple deserves accurate modelling and possibly compensation is that it may act as an excitation signal for the usually lightly damped dynamics of the two-mass system made up by the motor coupled with a load through an elastic transmission. As the torque ripple frequency is proportional to the motor

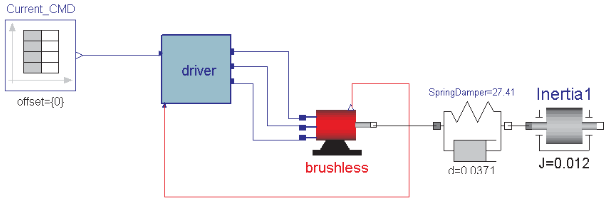

Figure 12: Model of the system including a load

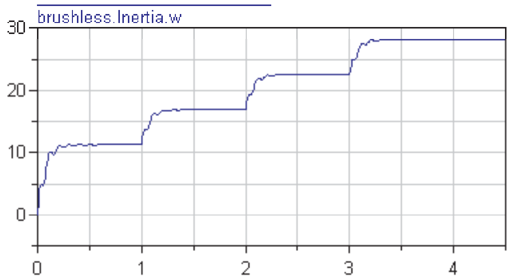

Figure 13: Motor velocity with a load, without ripple

velocity, this problem is particularly critical at those operating velocities when the multiple of the motor velocity is comparable to the natural frequency of the system. In order to confirm this analysis with simulation results, the model of the motor has been coupled to the models of an elastic transmission and a load, both taken from the Modelica.Mechanics.Rotational library (see Fig. 12).

The load has been given the same value as the inertia of the motor while the elastic parameter has been selected so as to have a resonance frequency approximately equal to $70 rad/s$. In a first simulation, four consecutive steps on the current command have been given, corresponding to 20%, 30%, 40% and 50% of the entire scale, in nominal conditions (i.e. with all the sources of ripple disabled). The result, in terms of the velocity of the motor is shown in Fig. 13, where the natural oscillations due to elasticity are evident, but also damped out by the natural damping on the system.

Then a ripple induced by the same offset on the current sensor as in the previous Section has been introduced. Notice that, as the natural frequency of the system is about $70 rad/s$, major problems to the system are expected when the average velocity of the motor is about one third ($23 rad/s$) of this value, namely in the third interval of the simulation. The result is confirmed in the plot of Fig. 14, where the effect of the matching between ripple frequency and natural frequency is most evident (once triggered in the third interval, the oscillations remains also in the fourth one).

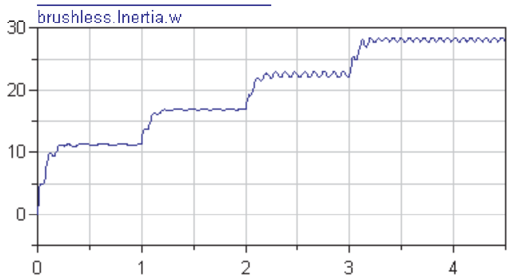

Figure 14: Motor velocity with a load, with ripple

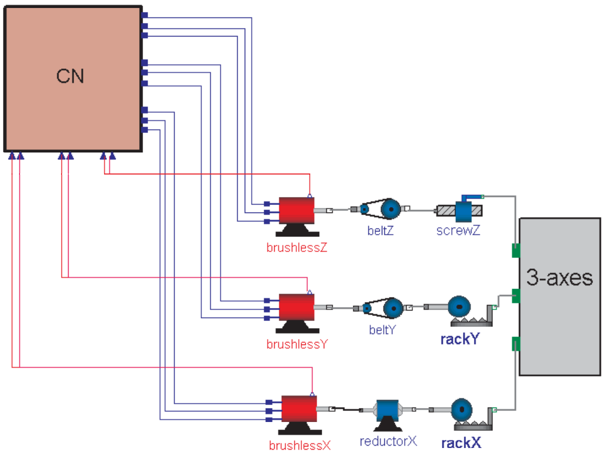

Figure 15: Top view of the simulator of a machining center

6 Use of the brushless motor in the simulation of a machining center

As already mentioned in the Introduction, the model of the brushless motor has been included in a library of elements used to simulate a complete machining center. Fig. 15 shows the top level of the simulator for a three axes machine. The model is composed of three parts: the simulation of the CN and the servodrive, entirely realized with the DYMOLA blocks, the simulation of the transmission chain for each axis, where the brushless motor has been used, and the simulation of the kinematic chain, realized with the blocks of the ModelicaAdditions.MultiBody library.

Again the multi-domain nature of DYMOLA and the physics driven assembly of the model turned out to be essential elements to fulfill the task, namely to realize a reliable simulation environment, easy to use for a non specialist of dynamic modelling.

7 Conclusions

DYMOLA proved to be a valuable tool to specify in the most natural way the model of the three phase brushless motor, in terms of a high index DAE system [1]. Simulations have been run to test various non nominal situations in brushless motors, where torque ripple can occur.

8 Acknowledgements

The support of student A. Samarani in building the model and performing simulations is acknowledged.

References

[1] Brenan, K.E., S.L. Campbell and L.R. Petzold: *Numerical solution of initial-value problems in differential algebraic equations*, North-Holland (1989).

[2] Cellier, F.: *Continuous system modelling*, Springer Verlag (1991).

[3] Ferretti, G., G. Magnani and P. Rocco: Modelling, identification and compensation of pulsating torque in permanent magnet ac motors. *IEEE Transactions on Industrial Electronics*, **46**, (1998), pp. 912–920.

[4] Jahns, T. M. and W. L. Soong: Pulsating torque minimization techniques for permanent magnet ac motor drives - a review. *IEEE Transactions on Industrial Electronics*, **43** (1996),pp. 321–330.

[5] Mattsson, S.E., H. Elmqvist and M. Otter: Physical system modeling with Modelica. *Control Engineering Practice*, **6**, (1998), pp. 501–510.

[6] Texas Instruments: Digital signal processing solution for permanent magnet synchronous motor. *Application Note. Literature Number: BPRA044*, (1997).

[7] Tiller, M.: *Introduction to physical modeling with Modelica*, Kluwer (2001).