Modelica™ - A Unified Object-Oriented Language for Physical Systems Modeling

LANGUAGE SPECIFICATION

Version 1.1

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Contents

1 Introduction

1.1 Overview of Modelica

Modelica is a language for modeling of physical systems, designed to support effective library development and model exchange. It is a modern language built on non-causal modeling with mathematical equations and object-oriented constructs to facilitate reuse of modeling knowledge.

1.2 Scope of the specification

The Modelica language is specified by means of a set of rules for translating a model described in Modelica to the corresponding model described as a flat hybrid DAE. The key issues of the translation (or instantiation in object-oriented terminology) are:

- Expansion of inherited base classes
- Parameterization of base classes, local classes and components
- Generation of connection equations from **connect** statements

The flat hybrid DAE form consists of:

- Declarations of variables with the appropriate basic types, prefixes and attributes, such as "parameter Real v=5".
- Equations from equation sections.
- Function invocations where an invocation is treated as a set of equations which are functions of all input and of all result variables (number of equations = number of basic result variables).
- Algorithm sections where every section is treated as a set of equations which are functions of the variables occuring in the algorithm section (number of equations = number of different assigned variables).
- When clauses where every when clause is treated as a set of conditionally evaluated equations, also called instantaneous equations, which are functions of the variables occuring in the clause (number of equations = number of different assigned variables).

Therefore, a flat hybrid DAE is seen as a set of equations where some of the equations are only conditionally evaluated (e.g. instantaneous equations are only evaluated when the corresponding when-condition becomes true).

The Modelica specification does not define the result of simulating a model or what constitutes a mathematically well-defined model.

1.3 Definitions and glossary

The semantic specification should be read together with the Modelica grammar. Non-normative text, i.e., examples and comments, are enclosed in *[ ]*, comments are set in *italics.*

Term	**Definition**
Component	An element defined by the production component-clause in the Modelica grammar (2.2.4).

Element	Class definitions, extends-clauses and component-clauses declared in a class.
Instantiation	The translation of a model described in Modelica to the corresponding model described as a hybrid DAE, involving expansion of inherited base classes, parameterization of base classes, local classes and components, and generation of connection equations from **connect** statements

2 Modelica syntax

2.1 Lexical conventions

The following syntactic meta symbols are used (extended BNF):

```
[ ]  optional
{ }  repeat zero or more times
```

The following lexical units are defined:

```
IDENT = NONDIGIT { DIGIT | NONDIGIT }
NONDIGIT = "_" | letters "a" to "z" | letters "A" to "Z"
STRING = """ { S-CHAR | S-ESCAPE } """
S-CHAR = any member of the source character set except double-quote """, or backslash "\"
S-ESCAPE = "\'" | "\"" | "\?" | "\\" |
           "\a" | "\b" | "\f" | "\n" | "\r" | "\t" | "\v"
DIGIT = 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
UNSIGNED_INTEGER = DIGIT { DIGIT }
UNSIGNED_NUMBER = UNSIGNED_INTEGER [ "." [ UNSIGNED_INTEGER ] ]
     [ ( e | E ) [ "+" | "-" ] UNSIGNED_INTEGER ]
```

Note: string constant concatenation "a" "b" becoming "ab" (as in C) is replaced by the "+" operator in Modelica.

Modelica uses the same comment syntax as C++ and Java. Inside a comment, the sequence `<HTML> .... </HTML>` indicates HTML code which may be used by tools to facilitate model documentation.

Keywords and built-in operators of the Modelica language are written in bold face.

2.2 Grammar

2.2.1 Model definition

```
model_definition:
   { [ final ] class_definition ";" }
```

2.2.2 Class definition

```
class_definition :
   [ partial ]
   ( class | model | record | block | connector | type |
     package | function )
   IDENT comment
   ( composition end IDENT |
     "=" name [ array_subscripts ] [ class_modification ] )

composition  :
   element_list
   { public element_list |
     protected element_list |
     equation_clause |
     algorithm_clause
   }
   [ external ]
```

```
element_list :
   { element ";" | annotation  ";" }

element :
   [ final ] ( [ replaceable ] class_definition | extends_clause |
               component_clause )
```

2.2.3 Extends

```
extends_clause :
   extends name [ class_modification ]
```

2.2.4 Component clause

```
component_clause:
   type_prefix type_specifier [ array_subscripts ] component_list

type_prefix :
   [ flow ] [ discrete | parameter | constant ] [ input | output ]

type_specifier :
   name

component_list :
   component_declaration { "," component_declaration }

component_declaration :
   declaration comment

declaration :
   IDENT [ array_subscripts ] [ modification ]
```

2.2.5 Modification

```
modification :
   class_modification [ "=" expression ]
 | "=" expression

class_modification :
   "(" { argument_list } ")"

argument_list :
   argument { "," argument }

argument :
   element_modification
 | element_redeclaration

element_modification :
   [ final ] component_reference modification

element_redeclaration :
   redeclare [ final ]
   ([ replaceable ] class_definition | extends_clause | component_clause1 )

component_clause1 :
   type_prefix type_specifier component_declaration
```

2.2.6 Equations

```
equation_clause :
  equation { equation ";" | annotation  ";" }

algorithm_clause :
   algorithm { algorithm ";" | annotation  ";" }

equation :
  ( simple_expression "=" expression
    | conditional_equation_e
    | for_clause_e
    | when_clause_e
    | connect_clause
    | assert_clause )
  comment

algorithm :
  ( component_reference ( ":=" expression | function_call )
    | conditional_equation_a
    | for_clause_a
    | while_clause
    | when_clause_a
    | assert_clause )
  comment

conditional_equation_e :
   if expression then
     { equation ";" }
   { elseif expression then
     { equation ";" }
   }
   [ else
     { equation ";" }
   ]
   end if

conditional_equation_a :
   if expression then
     { algorithm ";" }
   { elseif expression then
     { algorithm ";" }
   }
   [ else
     { algorithm ";" }
   ]
   end if

for_clause_e :
  for IDENT in expression loop
     { equation ";" }
  end for

for_clause_a :
  for IDENT in expression loop
     { algorithm ";" }
  end for
```

```
while_clause :
  while expression loop
    { algorithm ";" }
  end while

when_clause_e :
  when expression then
    { equation ";" }
  end when

when_clause_a :
  when expression then
    { algorithm ";" }
  end when

connect_clause :
  connect "(" connector_ref "," connector_ref ")"

connector_ref :
  IDENT [ array_subscripts ] [ "." IDENT [ array_subscripts ] ]

assert_clause :
  assert "(" expression "," STRING { "+" STRING } ")"
```

2.2.7 Expressions

```
expression :
   simple_expression
 | if expression then expression else expression

simple_expression :
   logical_expression [ ":" logical_expression [ ":" logical_expression ] ]

logical_expression :
   logical_term { or logical_term }

logical_term :
   logical_factor { and logical_factor }

logical_factor :
   [ not ] relation

relation :
   arithmetic_expression [ rel_op arithmetic_expression ]

rel_op :
   "<" | "<=" | ">" | ">=" | "==" | "<>"

arithmetic_expression :
   [ add_op ] term { add_op term }

add_op :
   "+" | "-"

term :
   factor { mul_op factor }
```

```
mul_op :
   "*" | "/"

factor :
   primary [ "^" primary ]

primary :
   UNSIGNED_NUMBER
 | STRING
 | false
 | true
 | component_reference [ function_call ]
 | "(" expression_list ")"
 | "[" expression_list { ";" expression_list } "]"
 | "{" expression_list "}"

name :
   IDENT [ "." name ]

component_reference :
   IDENT [ array_subscripts ] [ "." component_reference ]

function_call :
   "(" [ function_arguments ] ")"

function_arguments :
   expression_list
 | IDENT "=" expression { "," IDENT "=" expression }

expression_list :
   expression { "," expression }

array_subscripts :
   "[" subscript { "," subscript } "]"

subscript :
   ":" | expression

comment :
   [ STRING { "+" STRING } ] [ annotation ]

annotation :
   annotation class_modification
```

3 Modelica semantics

3.1 Fundamentals

Instantiation is made in a context which consists of an environment and an ordered set of parents.

3.1.1 Scoping and name lookup

3.1.1.1 Parents

The classes lexically enclosing an element form an ordered set of parents. A class defined inside another class definition (the parent) preceeds its enclosing class definition in this set.

Enclosing all class definitions is an unnamed parent which contains all top-level class definitions. The order of top-level class definitions in the unnamed parent is undefined.

During instantiation, the parent of an element being instantiated is a partially instantiated class. *[For example, this means that a declaration can refer to a name previously inherited through a previous extends clause.]*

[Example:

```
class C1 ... end C1;
class C2 ... end C2;
class C3
  Real x=3;
  C1 y;
  class C4
    Real z;
  end C4;
end C3;
```

The unnamed parent of class definition C3 contains C1 and C2 in arbitrary order. When instantiating class definition C3, the set of parents of the declaration of `x` is the partially instantiated class C3 followed by the unnamed parent with C1 and C2. The set of parents of `z` are C4, C3 and the unnamed parent in that order.]

3.1.1.2 Static name lookup

Names are looked up at class instantiation to find names of base classes, component types, etc.

For a simple name *[not composed using dot-notation]* lookup is performed as follows:

- When an element, equation or algorithm is instantiated, any name is looked up sequentially in each member of the ordered set of parents until a match is found.

For a composite name of the form A.B *[or A.B.C, etc.]* lookup is performed as follows:

- The first identifier *[A]* is looked up as defined above.
- If the identifier denotes a component, the rest of the name *[e.g., B or B.C]* is looked up in the component.
- If the identifier denotes a class, that class is temporarily instantiated with an empty environment and using the parents of the denoted class. The rest of the name *[e.g., B or B.C]* is looked up in the temporary instantiated class.

[The temporary class instantiation performed for composite names follow the same rules as class instantiation of the base class in an extends clause, local classes and the type in a component clause, except that the

environment is empty.]

All parts of a composite name denoting a component shall denote components. *[There are no class variables in Modelica.]*

3.1.2 Environment and modification

3.1.2.1 Environment

The environment contains arguments which modify elements of the class (e.g., parameter changes). The environment is built by merging class modifications, where outer modifications override inner modifications.

3.1.2.2 Merging of modifications

[The following larger example demonstrates several aspects:

```
class C1
  class C11
    parameter Real x;
  end C11;
end C1;
class C2
  class C21
    ...
  end C21;
end C2;
class C3
  extends C1;
  C11 t(x=3);        // ok, C11 has been inherited from C1
  C21 u;             // error, C21 has not yet been inherited
  extends C2;
end C3;
```

The environment of the declaration of t is (x=3). The environment is built by merging class modifications, as shown by:

```
class C1
  parameter Real a;
end C1;
class C2
  parameter Real b;
end C2;
class C3
  parameter Real x1;       // No default value
  parameter Real x2 = 2;   // Default value 2
  parameter C1 x3;         // No default value for x3.a
  parameter C1 x4(a=4);    // x4.a has default value 4
  extends C1;              // No default value for inherited element a
  extends C2(b=6);         // Inherited b has default value 6
end C3;
class C4
  extends C3(x2=22, x3(a=33), x4(a=44), C1(a=55), b=66);
end C4;
```

Outer modifications override inner modifications, e.g., `b=66` *overrides the nested class modification of* `extends C2(b=6)`*. This is known as merging of modifications:* `merge((b=66), (b=6))` *becomes* `(b=66)`*.*

An instantiation of class C4 will give an object with the following variables:

Variable	*Default value*

x1	*none*
x2	*22*
x3.a	*33*
x4.a	*44*
a	*55*
b	*66*

The last argument of the C3 modification shows that an inherited element (here, `b=66`) can be directly referred to, without specifying its base class as in `C1(a=55)`.]

3.1.2.3 Single modification

Two arguments of a modification shall not designate the same primitive attribute of an element. *[Example:*

```
class C1
  Real x[3];
end C1;
class C2 = C1(x=ones(3), x[2]=2);  // Error: x[2] designated twice
class C3
  class C4
    Real x;
  end C4;
  C4 a(x(unit = "V"), x = 5.0));
      // Ok, different attributes designated (unit and value)
end C3;
```

]

3.1.2.4 Instantiation order

The name of a declared element shall not have the same name as any other element in its partially instantiated parent class.

The elements of a class are instantiated in the order of declaration. An element is added to its partially instantiated parent class after the complete instantiation of the element. *[For example, `Real x = x;` is incorrect.]*

3.1.3 Subtyping and type equivalence

3.1.3.1 Subtyping of classes

For any classes S and C, S is a supertype of C and C is a subtype of S if they are equivalent or if:

- every public declaration element of S also exists in C (according to their names)
- those element types in S are supertypes of the corresponding element types in C.

A base class is the class referred to in an extends clause. The class containing the extends clause is called the derived class. *[Base classes of C are typically supertypes of C, but other classes not related by inheritance can also be supertypes of C.]*

3.1.3.2 Subtyping of components

Component B is subtype of A if:

- Both scalars or arrays with the same number of dimensions
- The type of B is subtype of the base type of A (base type for arrays)

- For every dimension of an array
  - The size of A is indefinite, or
  - The value of expression (size of B) - (size of A) is final constant equal to 0 (in the environment of B)

3.1.3.3 Type equivalence

Two types T and U are equivalent if:

- T and U denote the same built-in type (one of RealType, IntegerType, StringType or BooleanType), or
- T and U are classes, T and U contain the same public declaration elements (according to their names), and the elements types in T are equivalent to the corresponding element types in U.

3.1.3.4 Type identity

Two elements T and U are identical if:

- T and U are equivalent,
- they are either both declared as **final** or none is declared **final**,
- for a component their type prefixes are identical, and
- if T and U are classes, T and U contain the same public declaration elements (according to their names), and the elements in T are identical to the corresponding element in U.

3.1.4 Classes on external files

Class names are automatically mapped to a hierarchical structure of the operating system. Given that `A` denotes a class at global scope, the name path `A.B.C` is looked up as follows.

- If `A` is defined in the current translation unit, the rest of the path (`B.C`) is looked up inside `A`.
- Otherwise, `A` is located in an ordered list of library roots, called MODELICAPATH.

If the name `A` is a structured entity *[e.g. a directory]*, lookup of `B.C` progresses recursively in `A`.

If the name `A` is a non-structured entity *[e.g. a file]*, it shall contain only the complete definition of class `A`, and the rest of the path (`B.C`) is looked up inside that package. If the name A is a structured entity *[e.g. a directory with an optional node]*, the rest of the path is looked up in the node in the same way as in non-structured entity . If not found, lookup of B.C progresses recursively in A.

[In a file hierarchy, the node is stored in file `package.mo` in the package directory].

Otherwise the lookup fails.

[On a typical system, MODELICAPATH is an environment variable containing a semicolon-separated list of directory names. Classes are realized by directories with subdirectories, or files containing class definitions. The default file extension for Modelica is `.mo`; for example, the package `A` would be stored in file `A.mo`. If there is both a subdirectory `A` and a file `A.mo`, the lookup fails. Other forms of realizing packages are also possible, for example using a hierarchical database.]

[The first part of the path `A.B.C` (i.e., `A`) is located by searching the ordered list of roots in MODELICAPATH. If no root contains `A` the lookup fails. If `A` has been found in one of the roots, the rest of the path is located in `A`; if that fails, the entire lookup fails without searching for `A` in any of the remaining roots in MODELICAPATH.]

3.2 Declarations

3.2.1 Component clause

If the type specifier of the component denotes a built-in type (RealType, IntegerType, etc.), the instantiated component has the same type.

If the type specifier of the component does not denote a built-in type, the name of the type is looked up (3.1.1). The found type is instantiated with a new environment and the partially instantiated parent of the component. The new environment is the result of merging

- the modification of parent element-modification with the same name as the component
- the modification of the component declaration

in that order.

An environment that defines the value of a component of built-in type is said to define a declaration equation associated with the declared component. For declarations of vectors and matrices, declaration equations are associated with each element. *[This makes it possible to override the declaration equation for a single element in a parent modelification, which would not be possible if the declaration equation is regarded as a single matrix equation.]*

Array dimensions shall be non-negative parameter expressions.

Variables declared with the **flow** type prefix shall be a subtype of Real.

Components of function type may be instantiated. *[A modifier can be used to e.g. change parameters of the function. It is also possible to do such a modification with a class specialization.]* Components of a function are reinitialized with their start values at every function invocation.

3.2.2 Vectors, Matrices, and Arrays

3.2.2.1 Type declarations

The Modelica type system includes scalar number, vector, matrix (number of dimensions, ndim=2), and arrays of more than two dimensions. *[There is no distinguishing between a row and column vector.]*

The following table shows the two possible forms of declarations and defines the terminology. C is a placeholder for any class, including the builtin type classes Real, Integer, Boolean and String:

Modelica form 1	Modelica form 2	# dimensions	Designation	Explanation
C x;	C x;	0	Scalar	Scalar
C[n] x;	C x[n];	1	Vector	n - Vector
C[n, m] x;	C x[n, m];	2	Matrix	n x m Matrix
C[n, m, p, ...] x;	C x[m, n, p ,...];	k	Array	Array with k dimensions (k>=0).

[The number of dimensions and the dimensions sizes are part of the type, and shall be checked for example at redeclarations. Declaration form 1 displays clearly the type of an array, whereas declaration form 2 is the traditional way of array declarations in languages such as Fortran, C, C++ and is more general in some rare situations, e.g., when a square matrix with unknown sizes is declared which cannot be defined with the first form:

```
Real A[:,size(A,1)]; // square matrix of unknown size (size(A,1) is the size of the first dimension)
Real[:,size(A,1)] A; // error, because A is used before defined

Real[:]  v1, v2      // vectors v1 and v2 have unknown sizes. The actual sizes may be different.
```

It is possible to mix the two declaration forms, but it is not recommended

```
  Real[3,2] x[4,5];       // x has type  Real[4,5,3,2];
```
]

Zero-valued dimensions are allowed, so `C x[0];` declares an empty vector and `C x[0,3];` an empty matrix.

[Special cases:

Modelica form 1	Modelica form 2	# dimensions	Designation	Explanation
C[1] x;	C x[1];	1	Vector	1 – Vector, representing a scalar
C[1,1] x;	C x[1, 1];	2	Matrix	1 x 1 – Matrix, representing a scalar
C[n,1] x;	C x[n, 1];	2	Matrix	n x 1 – Matrix, representing a column
C[1,n] x;	C x[1, n];	2	Matrix	1 x n – Matrix, representing a row

]

The type of an array of array is the multidimensional array which is constructed by taking the first dimensions from the component declaration and subsequent dimensions from the maximally expanded component type. A type is maximally expanded, if it is either one of the built-in types (Real, Integer, Boolean, String) or it is not a type class. Before operator overloading is applied, a type class of a variable is maximally expanded.

[Example:

```
type Voltage = Real(unit = ”V”);
type Current = Real(unit = “A”);
connector Pin
  Voltage      v;       // type class of v = Voltage,  type of v = Real
  flow Current i;       // type class of  i = Current,  type of i = Real
end Pin;

type MultiPin = Pin[5];
MultiPin[4]  p;         // type class of p is MultiPin, type of p is Pin[4,5];

type Point = Real[3];

Point p1[10];

Real  p2[10,3];
```

The components pointvec1 and pointvec2 have identical types.

```
    p2[5] = p1[2]+ p2[4];      // equivalent to   p2[5,:] = p1[2,:] + p2[4,:]
    Real r[3] = p1[2];         // equivalent to   r[3] = p1[2,:]
```

]

[Automatic assertions at simulation time:

Let A be a declared array and i be the declared maximum dimension size of the d_i-dimension, then an assert statement “assert(i >= 0)” is generated provided this assertion cannot be checked at compile time.

Let A be a declared array and i be an index accessing an index of the d_i-dimension. Then for every such index-access an assert statement “assert(i >= 1 and i <= size(A,d_i))” is generated, provided this assertion cannot be checked at compile time.

For efficiency reasons, these implicit assert statement may be optionally suppressed.]

3.2.2.2 Built-in Functions for Array Expressions

The following function *cannot* be used in Modelica, but is utilized below to define other operators

promote(A,n)	Fills dimensions of size 1 from the right to array A upto dimension n, where "n >= ndims(A)" is required. Let C = promote(A,n), with nA=ndims(A), then ndims(C) = n,

	size(C,j) = size(A,j) for 1 <= j <= nA, size(C,j) = 1 for nA+1 <= j <= n, C[i_1, ..., i_nA, 1, ..., 1] = A[i_1, ..., i_nA]

[Function promote could not be used in Modelica, because the number of dimensions of the return array cannot be determined at compile time if n is a variable. Below, promote is only used for constant n].

The following built-in functions for array *expressions* are provided:

Modelica	*Explanation*
ndims(A)	Returns the number of dimensions k of array expression A, with k >= 0.
size(A,i)	Returns the size of dimension i of array expression A where i shall be > 0 and <= ndims(A).
size(A)	Returns a vector of length ndims(A) containing the dimension sizes of A.
scalar(A)	Returns the single element of array A. size(A,i) = 1 is required for 1 <= i <= ndims(A).
vector(A)	Returns a 1-vector, if A is a scalar and otherwise returns a vector containing all the elements of the array, provided there is at most one dimension size > 1.
matrix(A)	Returns promote(A,2), if A is a scalar or vector and otherwise returns the elements of the first two dimensions as a matrix. size(A,i) = 1 is required for 2 < i <= ndims(A).
transpose(A)	Permutes the first two dimensions of array A. It is an error, if array A does not have at least 2 dimensions.
outer(v1,v2)	Returns the outer product of vectors v1 and v2 (= matrix(v)*transpose(matrix(v))).
identity(n)	Returns the n x n Integer identity matrix, with ones on the diagonal and zeros at the other places.
diagonal(v)	Returns a square matrix with the elements of vector v on the diagonal and all other elements zero.
zeros(n_1,n_2,n_3,...)	Returns the n_1 x n_2 x n_3 x ... Integer array with all elements equal to zero (n_i >= 0).
ones(n_1,n_2,n_3,...)	Return the n_1 x n_2 x n_3 x ... Integer array with all elements equal to one (n_i >=0).
fill(s,n_1,n_2,n_3, ...)	Returns the n_1 x n_2 x n_3 x ... array with all elements equal to scalar expression s which has to be a subtype of Real, Integer, Boolean or String (n_i >= 0). The returned array has the same type as s.
linspace(x1,x2,n)	Returns a Real vector with n equally spaced elements, such that v=linspace(x1,x2,n), v[i] = x1 + (x2-x1)*(i-1)/(n-1) for 1 <= i <= n. It is required that n >= 2.
min(A)	Returns the smallest element of array expression A.
max(A)	Returns the largest element of array expression A.
sum(A)	Returns the sum of all the elements of array expression A.
product(A)	Returns the product of all the elements of array expression A.
symmetric(A)	Returns a matrix where the diagonal elements and the elements above the diagonal are identical to the corresponding elements of matrix A and where the elements below the diagonal are set equal to the elements above the diagonal of A, i.e., B := symmetric(A) -> B[i,j] := A[i,j], if i <= j, B[i,j] := A[j,i], if i > j.
cross(x,y)	Returns the cross product of the 3-vectors x and y, i.e. cross(x,y) = [x[2]*y[3]-x[3]*y[2]; x[3]*y[1]-x[1]*y[3]; x[1]*y[2]-x[2]*y[1]];
skew(x)	Returns the 3 x 3 skew symmetric matrix associated with a 3-vector, i.e., cross(x,y) = skew(x)*y; skew(x) = [0, -x[3], x[2]; x[3], 0, -x[1]; -x[2], x[1], 0];

[Example:

```
Real x[4,1,6];
size(x,1) = 4;
```

```
size(x);          // vector with elements 4, 1, 6
size(2*x+x ) = size(x);

Real[3] v1 = fill(1.0, 3);
Real[3,1] m = matrix(v1);
Real[3] v2 = vector(m);

Boolean check[3,4] = fill(true, 3, 4);
```

]

3.2.2.3 Vector, Matrix and Array Constructors

Array Construction

The constructor function **array**(A,B,C,...) constructs an array from its arguments according to the following rules:

- All arguments must have the same sizes, i.e., size(A) = size(B) = size(C) = ...
- All arguments must be *type equivalent.* The datatype of the result array is the maximally expanded type of the arguments. The maximally expanded types should be equivalent. Real and Integer subtypes can be mixed resulting in a Real result array where the Integer numbers have been transformed to Real numbers.
- Each application of this constructor function adds a one-sized dimension to the left in the result compared to the dimensions of the argument arrays, i.e., ndims(array(A,B,C)) = ndimes(A) + 1 = ndims(B) + 1, ...
- {A, B, C, ...} is a shorthand notation for array(A, B, C, ...).
- There must be at least one argument *[i.e., array() or {} is not defined]*.

[Examples:

{1,2,3} is a 3 vector of type Integer.
{ {11,12,13}, {21,22,23} } is a 2x3 matrix of type Integer
{{{1.0, 2.0, 3.0}}} is a 1x1x3 array of type Real.

```
Real[3] v = array(1, 2, 3.0);
type Angle = Real(unit="rad");
parameter Angle alpha = 2.0;     // type of alpha is Real.
array(alpha, 2, 3.0) is a 3 vector of type Real.
Angle[3] a = {1.0, alpha, 4};   // type of a is Real[3].
```

]

Array Concatenation

The function **cat**(k,A,B,C,...) concatenates arrays A,B,C,... along dimension k according to the following rules:

- Arrays A, B, C, ... must have the same number of dimensions, i.e., ndims(A) = ndims(B) = ...
- Arrays A, B, C, ... must be *type equivalen*t. The datatype of the result array is the maximally expanded type of the arguments. The maximally expanded types should be equivalent. Real and Integer subtypes can be mixed resulting in a Real result array where the Integer numbers have been transformed to Real numbers.
- k has to characterize an existing dimension, i.e., 1 <= k <= ndims(A) = ndims(B) = ndims(C).
- Arrays A, B, C, ... must have identical array sizes with the exception of the size of dimension k, i.e., size(A,j) = size(B,j), for 1 <= j <= ndims(A) and j <> k.

[Examples:

```
Real[2,3]  r1 =cat(1, {{1.0, 2.0, 3}}, {{4, 5, 6}});
Real[2,6]  r2  = cat(2, r1, 2*r1);
```

]

Concatenation is formally defined according to:

Let R = cat(k,A,B,C,...), and let n = ndims(A) = ndims(B) = ndims(C) = ..., then
size(R,k) = size(A,k) + size(B,k) + size(C,k) + ...
size(R,j) = size(A,j) = size(B,j) = size(C,j) = ..., for 1 <= j <= n and j <> k.
R[i_1, ..., i_k, ..., i_n] = A[i_1, ..., i_k, ..., i_n], for i_k <= size(A,k),
    R[i_1, ..., i_k, ..., i_n] = B[i_1, ..., i_k - size(A,i), ..., i_n], for i_k <= size(A,k) + size(B,k),
     ....
    where 1 <= i_j <= size(R,j) for 1 <= j <= n.

Array Concatenation along First and Second Dimensions

For convenience, a special syntax is supported for the concatenation along the first and second dimensions.

- *Concatenation along first dimension:*
  [A; B; C; ...] = cat(1, promote(A,n), promote(B,n), promote(C,n), ...) where
  n = max(2, ndims(A), ndims(B), ndims(C), ...). If necessary, 1-sized dimensions are added to the right of A, B, C before the operation is carried out, in order that the operands have the same number of dimensions which will be at least two.
- *Concatenation along second dimension:*
  [A, B, C, ...] = cat(2, promote(A,n), promote(B,n), promote(C,n), ...) where
  n = max(2, ndims(A), ndims(B), ndims(C), ...). If necessary, 1-sized dimensions are added to the right of A, B, C before the operation is carried out, especially that each operand has at least two dimensions.
- The two forms can be mixed. [...,...] has higher precedence than [...;...], e.g., [a, b; c, d] is parsed as [[a,b]; [c,d]].
- [A] = promote(A,max(2,ndims(A))), i.e., [A] = A, if A has 2 or more dimensions, and it is a matrix with the elements of A, if A is a scalar or a vector.
- There must be at least one argument (i.e. [] is not defined)

[Examples:

Real s1, s2, v1[n1], v2[n2], M1[m1,n], M2[m2,n], M3[n,m1], M4[n,m2], K1[m1,n,k], K2[m2,n,k];

[v1;v2] is a (n1+n2) x 1 matrix
[M1;M2] is a (m1+m2) x n matrix
[M3,M4] is a n x (m1+m2) matrix
[K1;K2] is a (m1+m2) x n x k array
[s1;s2] is a 2 x 1 matrix
[s1,s1] is a 1 x 2 matrix
[s1] is a 1 x 1 matrix
[v1] is a n1 x 1 matrix

Real[3] v1 = array(1, 2, 3);
Real[3] v2 = [{4, 5, 6}];
Real[3,2] m1 = [v1, v2];
Real[3,2] m2 = [v1, [4;5;6]]; // m1 = m2
Real[2,3] m3 = [1, 2, 3; 4, 5, 6];
Real[1,3] m4 = [1, 2, 3];
Real[3,1] m5 =[1; 2; 3];
]

Vector Construction

Vectors can be constructed with the general array constructor, e.g., `Real[3] v = {1,2,3}`.

The colon operator of *simple-expression* can be used instead of or in combination with this general constructor to construct Real and Integer vectors. Semantics of the colon operator:

- j : k is the Integer vector {j, j+1, ..., k}, if j and k are of type Integer.
- j : k is the Real vector {j, j+1.0, ... n}, with n = floor(k-j), if j and/or k are of type Real.
- j : k is an Integer vector with zero elements, if j > k.
- j : d : k is the Integer vector {j, j+d, ..., j+n*d}, with n = (k – j)/d, if j, d, and k are of type Integer.
- j : d : k is the Real vector {j, j+d, ..., j+n*d}, with n = floor((k-j)/d), if j, d, or k are of type Real.
- j : d : k is an Integer vector with zero elements, if d > 0 and j > k or if d < 0 and j < k.

[Examples:

```
Real v1[5] = 2.7 : 6.8;
Real v2[5] = {2.7, 3.7, 4.7, 5.7, 6.7}   // = same as v1
```

]

3.2.2.4 Array access operator

Elements of vector, matrix or array variables are accessed with `[]`. A colon is used to denote all indices of one dimension. A vector expression can be used to pick out selected rows, columns and elements of vectors, matrices, and arrays. The number of dimensions of the expression is reduced by the number of scalar index arguments.

[Examples:

- a[:, j] is a vector of the j-th column of a,
- a[j : k] is {[a[j], a[j+1], ... , a[k]},
- a[:,j : k] is [a[:,j], a[:,j+1], ... , a[:,k]],
- v[2:2:8] = v[{2,4,6,8}] .
- if `x` is a vector, `x[1]` is a scalar, but the slice `x[1:5]` is a vector (a vector-valued or colon index expression causes a vector to be returned).]

[Examples given the declaration x[n, m], v[k], z[i, j, p]:

Expression	# dimensions	Type of value
x[1, 1]	0	Scalar
x[:, 1]	1	n – Vector
x[1, :]	1	m – Vector
v[1:p]	1	p – Vector
x[1:p, :]	2	p x m – Matrix
x[1:1, :]	2	1 x m - "row" matrix
x[{1, 3, 5}, :]	2	3 x m – Matrix
x[: , v]	2	n x k – Matrix
z[: , 3, :]	2	i x p – Matrix
x[scalar([1]), :]	1	m – Vector
x[vector([1]), :]	1	1 x m - Matrix

]

3.2.2.5 Scalar, vector, matrix, and array operator functions

The mathematical operations defined on scalars, vectors, and matrices are the subject of linear algebra.

Numeric Type Class

The term *numeric class* is used below for a subtype of the Real or Integer type class.

Equality and Assignment of type classes

Equality "a=b" and assignment "a:=b" of scalars, vectors, matrices, and arrays is defined element-wise and require both objects to have the same number of dimensions and corresponding dimension sizes. The operands need to be type equivalent.

Type of a	Type of b	Result of a = b	Operation (j=1:n, k=1:m)
Scalar	Scalar	Scalar	a = b
Vector[n]	Vector[n]	Vector[n]	a[j] = b[j]
Matrix[n, m]	Matrix[n, m]	Matrix[n, m]	a[j, k] = b[j, k]
Array[n, m, …]	Array[n, m, …]	Array[n, m, …]	a[j, k, …] = b[j, k, …]

Addition and Subtraction of numeric type classes

Addition "a+b" and subtraction "a-b" of numeric scalars, vectors, matrices, and arrays is defined element-wise and require size(a) = size(b) and a numeric type class for a and b.

Type of a	Type of b	Result of a +/- b	Operation c := a +/- b (j=1:n, k=1:m)
Scalar	Scalar	Scalar	c := a +/- b
Vector[n]	Vector[n]	Vector[n]	c[j] := a[j] +/- b[j]
Matrix[n, m]	Matrix[n, m]	Matrix[n, m]	c[j, k] := a[j, k] +/- b[j, k]
Array[n, m, …]	Array[n, m, …]	Array[n, m, …]	c [j, k, …] := a[j, k, …] +/- b[j, k, …]

Scalar Multiplication of numeric type classes

Scalar multiplication "s*a" or "a*s" with numeric scalar s and numeric scalar, vector, matrix or array a is defined element-wise:

Type of s	Type of a	Type of s* a and a*s	Operation c := s*a or c := a*s (j=1:n, k=1:m)
Scalar	Scalar	Scalar	c := s * a
Scalar	Vector [n]	Vector [n]	c[j] := s* a[j]
Scalar	Matrix [n, m]	Matrix [n, m]	c[j, k] := s* a[j, k]
Scalar	Array[n, m, ...]	Array [n, m, ...]	c[j, k, ...] := s*a[j, k, ...]

Matrix Multiplication of numeric type classes

Multiplication "a*b" of numeric vectors and matrices is defined only for the following combinations:

Type of a	Type of b	Type of a* b	Operation c := a*b (j=1:size(a,1), k=1:size(a,2))
Vector [n]	Vector [n]	Scalar	c := sum_k(a[k]*b[k]), k=1:n
Vector [n]	Matrix [n, m]	Vector [m]	c[j] := sum_k(a[k]*b[k, j]), j=1:m, k=1:n
Matrix [n, m]	Vector [m]	Vector [n]	c[j] := sum_k(a[j, k]*b[k])

Matrix [n, m]	Matrix [m, p]	Matrix [n, p]	c[i, j] = sum_k(a[i, k]*b[k, j]), i=1:n, k=1:m, k=1:p

[Example:

```
Real A[3,3], x[3], b[3];
A*x = b;
x*A = b;                        // same as transpose([x])*A*b
[v]*transpose([v])              // outer product
v*M*v                           // scalar
tranpose([v])*M*v               // vector with one element
```

]

Scalar Division of numeric type classes

Division "a/s" of numeric scalars, vectors, matrices, or arrays a and numeric scalars s is defined element-wise.

Type of a	Type of s	Result of a / s	Operation c := a / s (j=1:n, k=1:m)
Scalar	Scalar	Scalar	c := a / s
Vector[n]	Scalar	Vector[n]	c[k] := a[k] / s
Matrix[n, m]	Scalar	Matrix[n, m]	c[j, k] := a[j, k] / s
Array[n, m, …]	Scalar	Array[n, m, …]	c[j, k, …] := a[j, k, …] / s

Exponentation of Scalars of numeric type classes

Exponentiation "a^b" is defined as `pow()` in the C language if both "a" and "b" are scalars of a numeric type class.

Scalar Exponentation of Square Matrices of numeric type classes

Exponentation "a^s" is defined if "a" is a square numeric matrix and "s" is a scalar as a subtype of Integer with s >= 0. The exponentation is done by repeated multiplication
(e.g. a^3 = a*a*a; a^0 = ones(size(a,1), size(a,2), ...); a^1 = a).

[Non-Integer exponents are forbidden, because this would require to compute the eigenvalues and eigenvectors of "a" and this is no longer an elementary operation].

Relational operators

Relational operators <, <=, >, >=, ==, <>, are only defined for *scalar* arguments. The result is Boolean and is *true* or *false* if the relation is fulfilled or not, respectively.

In relations of the form v1 == v2 or v1 <> v2, v1 or v2 shall not be a subtype of Real. *[The reason for this rule is that relations with Real arguments are transformed to state events (see section Events below) and this transformation becomes unnecessarily complicated for the == and <> relational operators (e.g. two crossing functions instead of one crossing function needed, epsilon strategy needed even at event instants). Furthermore, testing on equality of Real variables is questionable on machines where the number length in registers is different to number length in main memory]*.

Relations of the form "v1 rel_op v2", with v1 and v2 variables and rel_op a relational operator are called elementary relations. If either v1 or v2 or both variables are a subtype of Real, the relation is called a Real elementary relation.

Functions

Functions with one scalar return value can be applied to arrays element-wise, e.g. if A is a vector of reals, then sin(A) is a vector where each element is the result of applying the function sin to the corresponding element in A.

Consider the expression `f(arg1,...,argn)`, an application of the function f to the arguments arg1, ..., argn is defined.

For each passed argument, the type of the argument is checked against the type of the corresponding formal parameter of the function.

1. If the types match, nothing is done.
2. If the types do not match, and a type conversion can be applied, it is applied. Continued with step 1.
3. If the types do not match, and no type conversion is applicable, the passed argument type is checked to see if it is an n-dimensional array of the formal parameter type. If it is not, the function call is invalid. If it is, we call this a foreach argument.
4. For all foreach arguments, the number and sizes of dimensions must match. If they do not match, the function call is invalid.If no foreach arguments exists, the function is applied in the normal fashion, and the result has the type specified by the function definition.
6. The result of the function call expression is an n-dimensional array with the same dimension sizes as the foreach arguments. Each element ei,..,j is the result of applying f to arguments constructed from the original arguments in the following way.
   - If the argument is not a foreach argument, it is used as-is.
   - If the argument is a foreach argument, the element at index [i,...,j] is used.

If more than one argument is an array, all of them have to be the same size, and they are traversed in parallel.

[Examples:

```
sin({a, b, c})        = {sin(a), sin(b), sin(c)}    // argument is a vector
sin([a,b,c])          = [sin(a),sin(b),sin(c)]      // argument may be a matrix
atan({a,b,c},{d,e,f}) = {atan(a,d), atan(b,e), atan(c,f)}
```

This works even if the function is declared to take an array as one of its arguments. If pval is defined as a function that takes one argument that is a vector of Reals and returns a Real, then it can be used with an actual argument which is a two-dimensional array (a vector of vectors). The result type in this case will be a vector of Real.

```
pval([1,2;3,4]) = [pval([1,2]); pval([3,4])]
sin([1,2;3,4])  = [sin({1,2}); sin({3,4})]
                = [sin(1), sin(2); sin(3), sin(4)]
function Add
    input  Real e1, e2;
    output Real sum1;
algorithm
   sum1 := e1 + e2;
end Add;
```

Add(1, [1, 2, 3]) adds one to each of the elements of the second argument giving the result [2, 3, 4]. However, it is illegal to write 1 + [1, 2, 3], because the rules for the built-in operators are more restrictive.]

Empty Arrays

Arrays may have dimension sizes of 0. E.g.

```
Real x[0];                    // an empty vector
Real A[0, 3], B[5, 0], C[0, 0];   // empty matrices
```

- Empty matrices can be constructed with the fill function. E.g.

```
Real    A[:,:]    = fill(0.0, 0, 1)        // a Real 0 x 1 matrix
Boolean B[:, :, :] = fill(false, 0, 1, 0)   // a Boolean 0 x 1 x 0 matrix
```

- It is not possible to access an element of an empty matrix, e.g. v[j,k] is wrong if "v=[]" because the assertion fails that the index must be bigger than one.
- Size-requirements of operations, such as +, -, have also to be fulfilled if a dimension is zero. E.g.
  Real[3,0] A, B;
  Real[0,0] C;
  A + B // fine, result is an empty matrix
  A + C // error, sizes do not agree
- Multiplication of two empty matrices results in a zero matrix if the result matrix has no zero dimension sizes, i.e.,
  Real[0,m]*Real[m,n] = Real[0,n] (empty matrix)
  Real[m,n]*Real[n,0] = Real[m,0] (empty matrix)
  Real[m,0]*Real[0,n] = zeros(m,n) (non-empty matrix, with zero elements).

[Example:

```
Real u[p], x[n], y[q], A[n,n], B[n,p], C[q,n], D[q,p];
der(x) = A*x + B*u
     y = C*x + D*u
```

*Assume n=0, p>0, q>0: Results in "y = D*u"*

]

3.2.3 Short class definition

A class definition of the form

```
class IDENT1 = IDENT2 class_modification ;
```

is identical to the longer form

```
class IDENT1
  extends IDENT2 class_modification ;
end IDENT1;
```

A short class definition of the form

```
type TN = T[N] (optional modifier) ;
```

where N represents arbitrary array dimensions, conceptually yields an array class

```
array TN
  T[n] _ (optional modifiers);
end TN;
```

Such a array class has exactly one anonymous component (_). When a component of such an array class type is instantiated, the resulting instantiated component type is an array type with the same dimensions as _ and with the optional modifier applied.

[Example:

```
type Force = Real[3](unit={”Nm”,”Nm”,”Nm”});
Force f1;
Real f2[3](unit={”Nm”,”Nm”,”Nm”});
```

the types of `f1` and `f2` are identical.]

3.2.4 Local class definition

The local class is instantiated with the partially instantiated parent of the local class. The environment is the modification of any parent class element modification with the same name as the local class, or an empty environment.

The instantiated local class becomes an element of the instantiated parent class.

[The following example demonstrates parameterization of a local class:

```
class C1
  class Voltage = Real(unit="V");
  Voltage v1, v2;
end C1;
class C2
  extends C1(Voltage(unit="kV"));
end C2;
```

Instantiation of class C2 yields a local instance of class Voltage with unit "kV". The variables v1 and v2 thus have unit "kV".]

3.2.5 Extends clause

The name of the base class is looked up in the partially instantiated parent of the extends clause. The found base class is instantiated with a new environment and the partially instantiated parent of the extends clause. The new environment is the result of merging

- arguments of all parent environments that match names in the instantiated base class
- the modification of a parent element-modification with the same name as the base class
- the optional class modification of the extends clause

in that order.

[Examples of the three rules are given in the following example:

```
class A
  parameter Real a, b;
end A;
class B
  extends A(b=3);              // Rule #3
end B;
class C
  extends B(a=1, A(b=2));      // Rules #1 and #2
end C;
```

]

The elements of the instantiated base class become elements of the instantiated parent class.

[From the example above we get the following instantiated class:

```
class Cinstance
  parameter Real a=1;
  parameter Real b=2;
end Cinstance;
```

The ordering of the merging rules ensures that, given classes A and B defined above,

```
class C2
  B bcomp(b=1, A(b=2));
end C2;
```

yields an instance with `bcomp.b=1`, which overrides `b=2`.]

The declaration elements of the instantiated base class shall either

- Not already exist in the partially instantiated parent class *[i.e., have different names]* .
- Be identical to any element of the instantiated parent class with the same name and the same level of protection (**public** or **protected**). In this case, the element of the instantiated base class is ignored.

Otherwise the model is incorrect.

[The second rule says that if an element is inherited multiple times, the first inherited element overrides later inherited elements:

```
class A
  parameter Real a, b;
end A;

class B
  extends A(a=1);
  extends A(b=2);
end B;
```

Class B is well-formed and yields an instantiated object with elements `a` and `b` inherited from the first extends clause:

```
class Binstance
  parameter Real a=1;
  parameter Real b;
end Binstance;
```

]

Equations of the instantiated base class that are syntactically equivalent to equations in the instantiated parent class are discarded. *[Note: equations that are mathematically equivalent but not syntactically equivalent are not discarded, hence yield an overdetermined system of equations.]*

3.2.6 Redeclaration

A **redeclare** construct replaces the declaration of an extends clause, local class or component in the modified element with another declaration. The type specified in the redeclaration shall be a subtype of the type in the original declaration.

The element modifications of the redeclaration and the original declaration are merged in the usual way.

[Example:

```
class A
  parameter Real x;
end A;
class B
  parameter Real x=3.14, y;    // B is a subtype of A
end B;
class C
  A a(x=1);
end C;
class D
  extends C(redeclare B a(y=2));
end D;
```

which effectively yields a class D2 with the contents

```
class D2
  B a(x=1, y=2);
end D2;
```

]

The following additional constraints apply to redeclarations:

- an element declared as **final** cannot be redeclared
- an element declared as **constant** can only be redeclared with **constant** or **final constant**
- an element declared as **parameter** can only be redeclared with **parameter**, **constant** or **final constant**
- an element declared as **discrete** can only be redeclared with **discrete**, **parameter**, **constant** or **final**

constant

- a **function** can only be redeclared as **function**
- an element declared as **flow** can only be redeclared with **flow**
- an element declared as not **flow** can only be redeclared without **flow**

Modelica does not allow a protected element to be redeclared as public, or a public element to be redeclared as protected.

A scalar may be redeclared as a matrix. A matrix may be redeclared as a scalar. Matrix dimensions may be redeclared.

3.3 Equations

3.3.1 Equation clause

The instantiated equation is identical to the non-instantiated equation.

Names in an equation shall be found by looking up in the partially instantiated parent of the equation.

Equation equality **=** shall not be used in an algorithm clause. The assignment operator **:=** shall not be used in an equation clause.

3.3.2 For clause

The expression of a for clause shall be a vector expression. It is evaluated once for each for clause. In an equation section, the expression of a for clause shall be a parameter expression.

[Example:

```
for i in 1:10 loop              // i takes the values 1,2,3,...,10
for r in 1.0 : 1.5 : 5.5 loop   // r takes the values 1.0, 2.5, 4.0, 5.5
for i in {1,3,6,7} loop         // i takes the values 1, 3, 6, 7
```

]

3.3.3 When clause

The expression of a when clause shall be a Boolean scalar or vector expression. The equations within a when clause are activated when the scalar or any one of the elements of the vector expression becomes true. A when clause shall not be used within a function class.

[Example:

Equations are activated when x becomes > 2:

```
when x > 2 then
  y1 = 2*x + y2;
  y2 = sin(x);
end when;
```

Equations are activated when either x becomes > 2 or sample(0,2) becomes true or x becomes less than 5:

```
when {x > 2, sample(0,2), x < 5} then
  y1 = 2*x + y2;
  y2 = sin(x);
end when;
```

The equations in a when clause are sorted independently form each other with all other equations.]

A when clause

```
when {condition1, condition2, ..., conditionN} then
  ...
end when;
```

is equivalent to the following special if-clause

```
if edge(condition1) or edge(condition2) or ... edge(conditionN) then
  ...
end if;
```

with "`edge(A)= A and not pre(A)`" and the additional guarantee, that the equations within this special if clause are only evaluated at event instants.

When clauses cannot be nested.

[Example:

The following when clause is invalid:

```
when x > 2 then
  when y1 > 3 then
    y2 = sin(x);
  end when;
end when;
```

]

The expression of an assert clause shall evaluate to true. *[The intent is to perform a test of model validity and to report the failed assertion to the user if the expression evaluates to false. The means of reporting a failed assertion are dependent on the simulation environment.]*

3.3.4 Connections

Connections between objects are introduced by the **connect** statement in the equation part of a class. The **connect** construct takes two references to connectors, each of which is either an element of the same class as the **connect** statement (an outer connector) or an element of one of its components (an inner connector). The two main tasks are to:

- Build connection sets from **connect** statements.
- Generate equations for the complete model.

Definitions:

Connection sets

> A connection set is a set of variables connected by means of connect clause. A connection set shall contain either only flow variables or only non-flow variables.

Inner and outer connectors

> In a class M, each connector element of that class is called an outer connector with respect to M. Each connector element of some element of M is called an inner connector with respect to M.

3.3.4.1 Generation of connection equations

For every use of the connect statement

```
connect(a, b);
```

the primitive components of `a` and `b` form a connection set. If any of them already occur in a connection set from previous connects in the same class or nested components, these sets are merged to form one connection set. Composite connector types are broken down into primitive components. Each connection set is used to generate equations for across and through (zero-sum) variables of the form

```
a1 = a2 = ... = an;
```

```
z1 + z2 + (-z3) + ... + zn = 0;
```

In order to generate equations for through variables *[using the `flow` prefix]*, the sign used for the connector variable z_i above is +1 for inner connectors and -1 for outer connectors *[`z3` in the example above]*.

For each unconnected through (zero-sum) variable the following equation is implicitly generated:

```
z = 0;
```

3.3.4.2 Restrictions

A component of an inner connector declared with the **input** type prefix shall not occur in more than one **connect** statement in that scope. A component of an outer connector declared with the **output** type prefix shall not occur in more than one **connect** statement in that scope. Two components declared with the **input** type prefix shall not be connected in the same scope. Two components declared with the **output** type prefix shall not be connected in the same scope.

Subscripts in a connector reference shall be constant expressions.

If the array sizes do not match, the original variables are filled with one-sized dimensions from the left until the number of dimensions match before the connection set equations are generated.

Constants or parameters in connected components yield the appropriate assert statements; connections are not generated.

3.4 Functions

There are two forms of function application, see section 2.2.7. In the first form,

```
f(3.5, 5.76)
```

the arguments are associated with the *[formal]* parameters according to their position in the argument list. Thus argument *i* is passed to parameter *i*, where the order of the parameters is given by the order of the component declarations in the function definition. The first input component is parameter number 1, the second input component is parameter number 2, and so on. When a function is called in this way, the number of arguments and parameters must be the same.

In the second form of function application,

```
g(x=3.5, y=5.76)
```

the parameters are explicitly associated with the arguments by means of equations in the argument list. Parameters which have default values need not be specified in the argument list.

The type of each argument must agree with the type of the corresponding parameter, except where the standard type coercions can be used to make the types agree. (See also section 3.2.2.5 on applying scalar functions to arrays.)

[Example. Suppose a function f is defined as follows:

```
function f
   input Real x;
   input Real y;
   input Real z = 10.0;
   output Real r;
   ...
end f;
```

Then the following two applications are equivalent:

```
f(1.0, 2.0, 10.0)
f(y = 2.0, x = 1.0)
```

]

A function may have more than one output component, corresponding to multiple return values. When a function

has a single return value, a function application is an expression whose value and type are given by the value and type of the output component.

The only way to call a function having more than one output component is to make the function call the RHS of an equation or assignment. In these cases, the LHS of of the equation or assignment must be a list of component references within parentheses. The component references are associated with the output components according to their position in the list. Thus output component i is set equal to, or assigned to, component reference i in the list, where the order of the output components is given by the order of the component declarations in the function definition.

The number of component references in the list must agree with the number of output components.

The type of each output parameter must agree with the type of the corresponding component references in the list on the LHS.

[Example. Suppose a function f is defined as follows:

```
function f
   input Real x;
   input Real y;
   output Real r1;
   output Real r2;
   output Real r3;
   ...
end f;
```

Then the following equation and assignment show the two possible ways of calling f:

```
   (x, y, z) = f(1.0, 2.0);
   (x, y, z) := f(1.0, 2.0);
```

]

The only permissible use of an expression in the form of a list of expressions in parentheses, is when it is used as the LHS of an equation or assignment where the RHS is an application of a function with more than one output component. In this case, the expressions in the list shall be component references.

[Example. The following are illegal:

```
   (x+1, 3.0, z/y) = f(1.0, 2.0);     // Not a list of component references.
   (x, y, z) + (u, v, w)              // Not LHS of suitable eqn/assignment.
```

]

3.5 Variability constraints

The prefixes **discrete**, **parameter**, **constant** of a component declaration define in which situation the value of a variable can be changed, as outlined in the following table:

	public	*experiment*	*at event*	*during integration*
no prefix	variable	variable	variable	variable
discrete	variable	variable	variable	fixed
parameter	variable	variable	fixed	fixed
constant	variable	fixed	fixed	fixed
final constant	fixed	fixed	fixed	fixed

The column heading "**public**" characterizes a declaration in a public section of a model. "*Experiment*" characterizes an environment in which the simulation experiment is defined. Here, only data of the model can be changed but no other model properties, such as equations. "*At event*" characterizes an event instant of a simulation. The last column characterizes the period of continuous integration of a simulation.

*[A **discrete** component is a piecewise constant signal which changes its values only at event instants during simulation. This prefix is needed in order that special algorithms, such as the algorithm of Pantelides for index reduction, can be applied (it must be known that the time derivative of these variables is identical to zero). Furthermore, memory requirements can be reduced in the simulation environment, if it is known that a component can only change at event instants.*

*A **parameter** component is constant during simulation. This prefix gives the library designer the possibility to express that the physical equations in a library are only valid if some of the used components are constant during simulation. The same also holds for the **discrete** and **constant** prefix. Additionally, the **parameter** prefix allows a convenient graphical user interface in the experiment environment, to support quick changes of the most important constants of a model.*

*A **constant** component is a constant which can be changed by a constant model modifier(the modifier needs to be a constant expression), e.g., when instantiating a class, and which cannot be changed in an experiment environment. This allows for example to hide constants from the experiment environment to reduce the visible parameters.*
In combination with an if-clause, this allows to remove parts of a model before the symbolic processing of a model takes place in order to avoid variable causalities in the model (similar to #ifdef in C). Class parameters can be sometimes used as an alternative. Example:

```
model Inertia
   constant Boolean state = true;
   ...
equation
   J*a = t1 - t2;
   if state then     // code which is removed at compile-time,
      der(v) = a;    // if state=false
      der(r) = v;
   end if
end Inertia;
```

]

Constant expressions are:

- Real, integer, boolean and string literals.
- Real, integer, boolean and string variables declared as **constant** or **final constant**.
- Logical expressions with constant subexpressions not containing function calls.
- Logical expressions with constant subexpressions using the functions **div**, **rem**, **ceil**, **floor**, **integer**, **size**, **ndims**, **min**, **max**, **abs**, **sign**.

Parameter expressions are:

- Constant expressions.
- Real, integer, boolean and string variables declared as **parameter**.
- Logical expressions with parameter subexpressions not containing function calls.
- Logical expressions with parameter subexpressions using the functions **div**, **rem**, **ceil**, **floor**, **integer**, **size**, **ndims**, **min**, **max**, **abs**, **sign**.

Discrete expressions are:

- Parameter expressions.
- Real, integer, boolean and string variables declared as discrete.
- Function calls where all input arguments of the function are declared as discrete, parameter or constant.
- Expressions in the body of a when clause.

- Logical expressions with discrete subexpressions.

For causality analysis or sorting, components declared as **discrete**, **parameter** or **constant** are assumed to be *known*.

Components declared as **constant** shall have an associated declaration equation. *[If the declaration equation is not defined in the declaration itself, all instances of that declaration must be given a modification that defines the declaration equation.]*

Before the initial event takes place, components declared as **discrete** are set equal to their start value.

A component declared with the prefix **final constant** shall not be redeclared and its value shall not be modified. *[In general, **final** `type` defines that no redeclaration of `type` is possible, but is not associated with the variable value.]*

If the declaration or the modification of a **parameter** variable contains a defining equation for the parameter and this equation references directly or indirectly **parameter**, **discrete** or non-constant variables, then this parameter is *not visible* in an experiment environment.

[Example:

```
model modelA
   public
      parameter Real p1 = 1;
      parameter Real p2 = 1 + p1;
      constant  Real p3 = 1 + p1;
      test obj(final p4 = 1 + p1,    // parameter Real p4
                     p5 = 1 + p1,    // parameter Real p5
   protected
      parameter Real p6 = 1 + p1;
      constant  Real p7 = 1 + p1;
end modelA;

model modelB
   modelA a(p1=3, p2=4+a.p1, p3=4+a.p1, obj.p5=4+a.p1);
   ...
end model1B
```

*When instantiating model `modelA`, only variables `p1,p2,p3,p5` are visible and can be modified. In an experiment environment only `p1` is visible because all other parameters are functions of `p1`. If `p1=5` is set, `p2=p5=9`, `p3=7`, `p4=p6=6`, and `p7=4`. Note, that the **constant** variables are already fixed at compile time using the default value for `p1`.]*

A set of **parameter** components can be computed from a set of other **parameter** components by using a function call.

[Example:

```
function fc  // or use an external function
   output Real p3, p4;
   input  Real p1, p2;
algorithm
   p3 := p1*p2;
   p4 := p3*p1 + p2;
end fc;

model Test
   parameter Real p1=2, p2=3;
   ...
end Test;
```

]

3.6 Events and Synchronization

The integration is halted and an event occurs whenever a Real elementary relation, e.g. "x > 2", changes its value. During continuous integration the value of a relation is *constant*. The value of a relation can only be changed at event instants *[in other words, Real elementary relations induce state or time events]*. At an event instant, a relation is taken literally. During continuous integration a Real elementary relation has the constant value of the relation from the last event instant.

[Example:

```
y = if u > uMax then uMax else if u < uMin then uMin else u;
```

During continuous integration always the same if branch is evaluated. The integration is halted whenever u-uMax or u-uMin crosses zero. At the event instant, the correct if-branch is selected and the integration is restarted.

Numerical integration methods of order n (n>=1) require continuous model equations which are differentiable upto order n. This requirement can be fulfilled if Real elementary relations are not treated literally but as defined above, because discontinuous changes can only occur at event instants and no longer during continuous integration.]

[It is a quality of implementation issue that the following special relations

`time >=` *discrete expression*
`time <=` *discrete expression*
`time >` *discrete expression*
`time <` *discrete epxression*

trigger a time event at "time = discrete expression", i.e., the event instant is known in advance and no iteration is needed to find the exact event instant.]

Relations are taken literally also during continuous integration, if the relation or the expression in which the relation is present, are the argument of the **noEvent**(..) function. The noEvent feature is propagated to all subrelations in the scope of the noEvent function.

[Example:

```
y = noEvent( if u > uMax then uMax else if u < uMin then uMin else u);
```

The if-expression is taken literally without inducing state events.

*The **noEvent** function is useful, if e.g. the modeller can guarantee that the used if-clauses fulfill at least the continuity requirement of integrators. In this case the simulation speed is improved, since no state event iterations occur during integration. Furthermore, the **noEvent** function is used to guard agains "outside domain" errors, e.g. y = if **noEvent**(x >= 0) then sqrt(x) else 0.]*

All equations and assignment statements within *when clauses* and all assignment statements within *function classes* are implicitly treated with the **noEvent** function, i.e., relations within the scope of these operators never induce state or time events. *[Using state events in when-clauses is unnecessary because the body of a when clause is not evaluated during continuous integration.]*

It is not allowed to use the **noEvent** function directly or indirectly for relations in discrete expressions outside of when clauses or relations in the conditions of when clauses *[otherwise it cannot be guaranteed that discrete expressions and when clauses are only evaluated at event instants]*.

[Example:

```
Limit1 = noEvent(x1 > 1);
Limit2 = x2 > 10;
Limit = Limit1 or Limit2;
when Limit then
   Close = true;
end when;
```

This is an error, because Limit1 is indirectly used in the condition of a when clause and therefore the body of the

when clause may be evaluated during continuous integration which can lead to a discontinuous change in the model equations.

```
  discrete Boolean off1;
  Boolean          off2;
equation
  off1 = s1 < 0;            // noEvent(s1 < 0) is not allowed
  off2 = noEvent(s2 < 0)    // possible, because no discrete variable
  u1 = if off1 then s1 else 0;
  u2 = if off2 then s2 else 0;
```

*Since off1 is declared as **discrete**, it is guaranteed that off1 can only change its values at event instants. Variable off2 may change its value during continuous integration. As a result, u1 is guaranteed to be continuous during continuous integration whereas no such guarantee exists for u2.]*

Modelica is based on the synchronous data flow principle which is defined in the following way:

1. All Variables keep their actual values until these values are explicitly changed. Variable values can be accessed at any time instant during continuous integration and at event instants.
2. At every time instant, during continuous integration and at event instants, the active equations express relations between variables which have to be fulfilled *concurrently* (equations are not active if the corresponding if-branch, when-clause or block in which the equation is present is not active).
3. Computation and communication at an event instant does not take time*. [If computation or communication time has to be simulated, this property has to be explicitly modeled]*.
4. The total number of equations is identical to the total number of unknown variables (= single assignment rule).

[These rules guarantee that variables are always defined by a unique set of equations. It is not possible that a variable is e.g. defined by two equations, which would give rise to conflicts or non-deterministic behaviour. Furthermore, the continuous and the discrete parts of a model are always automatically „synchronized“. Example:

```
when condition1 then
   close = true;
end when;

when condition2 then
   close = false;
end when;
```

This is not a valid model because rule 4 is violated since there are two equations for the single unknown variable close. If this would be a valid model, a conflict occurs when both conditions become true at the same time instant, since no priorities between the two equations are assigned. To become valid, the model has to be changed to:

```
when {condition1, condition2} then
   close = if edge(condition1) then true else false;
end when;
```

Here, it is well-defined if both conditions become true at the same time instant (condition1 has a higher priority than condition2).]

There is no guarantee that two different events occur at the same time instant.

[As a consequence, synchronization of events has to be explicitly programmed in the model, e.g. via counters. Example:

```
   Boolean fastSample, slowSample;
   Integer ticks(start=0);
equation
   fastSample = sample(0,1);
```

```
when fastSample then
   ticks      = if pre(ticks) < 5 then pre(ticks)+1 else 0;
   slowSample = pre(ticks) == 0;
end when;

when fastSample then   // fast sampling
   ...
end when;

when slowSample then   // slow sampling (5-times slower)
   ...
end when;
```

The slowSample when-clause is evaluated at every 5th occurrence of the fastSample when clause.]

[The single assignment rule and the requirement to explicitly program the synchronization of events allow a certain degree of model verification already at compile time. For example, "deadlock" between different when-clauses is present if there are algebraic loops between the equations of the when-clauses.]

3.7 Restricted classes

The keyword **class** can be replaced by one of the following keywords: **record**, **type**, **connector**, **model**, **block**, **package** or **function**. Certain restrictions will then be imposed on the content of such a definition. The following table summarizes the restrictions.

record	No equations are allowed in the definition or in any of its components. May not be used in connections.
type	May only be extension to the predefined types, records or matrix of type.
connector	No equations are allowed in the definition or in any of its components.
model	May not be used in connections.
block	Fixed causality, input-output block. Each component of an interface must either have Causality equal to Input or Output. May not be used in connections.
package	May only contain declarations of classes and constants.
function	Same restrictions as for block. Additional restrictions: no equations, only one algorithm section.

3.8 Variable attributes

The attributes of the predefined variable types are described below with Modelica syntax although they are predefined; redeclaration of any of these types is an error. The definitions use RealType, IntegerType, BooleanType and StringType as mnemonics correponding to machine representations. *[Hence the only way to declare a subtype of e.g. Real is to use the extends mechanism.]*

```
type Real
  RealType value;                       // Accessed without dot-notation
  parameter StringType  quantity    = "";
  parameter StringType  unit        = ""  "Unit used in equations";
  parameter StringType  displayUnit = ""  "Default display unit";
  parameter RealType    min=-Inf, max=+Inf;  // Inf denotes a large value
  parameter RealType    start = 0;           // Initial value

equation
  assert(value = min and value <= max, "Variable value out of limit");
end Real;

type Integer
```

```
  IntegerType value;                    // Accessed without dot-notation
  parameter IntegerType min=-Inf max=+Inf;
  parameter IntegerType start = 0;      // Initial value

equation
  assert(value = min and value <= max, "Variable value out of limit");
end Integer;

type Boolean
  BooleanType value;                   // Accessed without dot-notation
  parameter BooleanType start = false; // Initial value
end Boolean;

type String
  StringType value;                   // Accessed without dot-notation
  parameter StringType start = "";    // Initial value
end String;
```

[For external functions in C, RealType by default maps to `double` and IntegerType by default maps to `int`. In the mapping proposed in Annex F of the future C9X standard, RealType/double matches the IEC 60559:1989 (ANSI/IEEE 754-1985) double format. Typically IntegerType represents a 32-bit 2-complement signed integer.]

3.9 Intrinsic library functionality

3.9.1 Built-in variable time

All declared variables are functions of the independent variable **time**. Time is a built-in variable available in all classes, which is treated as an input variable. It is implicitly defined as:

```
input Real time (final quantity = "Time",
                 final unit     = "s");
```

The value of the **start** attribute of time is set to the time instant at which the simulation is started.

[Example:
Trigger an event at start time + 10 s:

```
parameter Real T0 = time.start + 10;
when time >= T0 then
   ...
end when;
```

]

3.9.2 Modelica built-in operators

Built-in operators of Modelica have the same syntax as a function call. However, they do **not** behave as a mathematical function, because the result depends not only on the input arguments but also on the status of the simulation. The following operators are supported:

der(x)	The time derivative of x. Variable x need to be a (non-discrete) subtype of Real. If x is an array, the operator is applied to all elements of the array.
initial()	Returns true at the initial time instant.
terminal()	Returns true at a regular halt of the simulation, but not in the case of an error stop.
noEvent(expr)	Real elementary relations within expr are taken literally, i.e., no state or time event is triggered.
sample(start,interval)	Returns true and triggers time events at time instants `"start +`

	`i*interval" (i=0,1,...)`. During continuous integration the operator returns always false. The starting time "start" and the sample interval "interval" need to be parameter expressions and need to be a subtype of Real or Integer.
pre(y)	Returns the "left limit" $y(t^{pre})$ of variable y(t) at a time instant t. At an event instant, $y(t^{pre})$ is the value of y after the last event iteration at time instant t (see comment below). The **pre** operator can be applied if the following three conditions are fulfilled simultaneously: (a) variable y is a subtype of Boolean, Integer or Real, (b) the operator is applied within a **when** clause or y is declared as **discrete**, (c) the operator is **not** applied in a **function** class.
edge(b)	Is expanded into "(b **and not pre**(b))" for Boolean variable b. The same restrictions as for the **pre** operator apply (e.g. not to be used in function classes).
reinit(x, expr)	Reinitializes state variable x with expr at an event instant. Argument x need to be (a) a subtype of Real and (b) the **der**-operator need to be applied to it. expr need to be an Integer or Real expression. The reinit operator can only be applied once for the same variable x.
abs(v)	Is expanded into "(**if** v >= 0 **then** v **else** –v)". Argument v needs to be an Integer or Real expression. *[Note, outside of a when clause state events are triggered]*.
sign(v)	Is expanded into "(**if** v > 0 **then** 1 **else if** v < 0 **then** –1 **else** 0)". Argument v needs to be an Integer or Real expression. *[Note, outside of a when clause state events are triggered]*
sqrt(v)	Returns "**if noEvent**(v >= 0) **then** squareRoot(x) **else** OutsideDomainError". Argument v needs to be an Integer or Real expression.
div(x,y)	Returns the algebraic quotient `x/y` with any fractional part discarded (also known as truncation toward zero). Result and arguments shall have type Real. [Note: this is defined for / in C9X; in Standard C the result for negative numbers is implementation-defined, so the standard function `div()` must be used.]. The input arguments need to be discrete expressions.
rem(x,y)	Returns the integer remainder of `x/y`, such that `div(x,y) * y + rem(x, y) = x`. Result and arguments shall have type Real. The input arguments need to be discrete expressions.
ceil(x)	Returns the smallest integer not less than `x`. Result and argument shall have type Real. The input argument needs to be a discrete expression.
floor(x)	Returns the largest integer not greater than `x`. Result and argument shall have type Real. The input argument needs to be a discrete expression.
integer(x)	Returns the largest integer not greater than `x`. The argument shall have type Real. The result has type Integer. The input argument needs to be a discrete expression.
delay(expr,delayTime,delayMax) **delay**(expr,delayTime)	Returns `"expr(time – delayTime)"` for `time > time.start + delayTime` and `"expr(time.start)"` for `time <= time.start + delayTime`. The arguments, i.e., expr, delayTime and delayMax, need to be subtypes of Real. `delayMax` needs to be additionally a parameter expression. The following relation shall hold: `0 <= delayTime <= delayMax.` If `delayMax` is not supplied in the argument list, `delayTime` need to be a parameter expression.

A new event is triggered if at least for one variable v "**pre**(v) <> v" after the active model equations are evaluated at an event instant. In this case the model is at once reevaluated. This evaluation sequence is called

"*event iteration*". The integration is restarted, if for all v used in **pre**-operators the following condition holds: "**pre**(v) == v".

*[If v and **pre**(v) are only used in when clauses, the translator might mask event iteration for variable v since v cannot change during event iteration. It is a "quality of implementation" to find the minimal loops for event iteration, i.e., not all parts of the model need to be reevaluated.*
The language allows that mixed algebraic systems of equations occur where the unknown variables are of type Real, Integer or Boolean. These systems of equations can be solved by a global fix point iteration scheme, similarily to the event iteration, by fixing the Boolean and Integer unknowns during one iteration. Again, it is a quality of implementation to solve these systems more efficiently, e.g., by applying the fix point iteration scheme to a subset of the model equations.]

The **reinit** operator does not break the single assignment rule, because **reinit**(x,expr) makes the previously known state variable x unknown and introduces the equation "x = expr".

*[If a higher index system is present, i.e. constraints between state variables, some state variables need to be redefined to non-state variables. If possible, non-state variables should be chosen in such a way that states with an applied **reinit** operator are not utilized. If this is not possible, an error occurs, because the reinit operator is applied on a non-state variable.*

*Examples for the usage of the **reinit** operator:*

Bouncing ball:

```
der(h) = v;
der(v) = -g;
when h < 0 then
  reinit(v, -e*v);
end when;
```

Self-initializing block:

```
block PT1 "first order filter"
  parameter Real T "time constant";
  parameter Real k "gain";
  input  Real u;
  output Real y;
protected
  Real x;
equation
  der(x) = (u - x) / T;
      y  = k*x;
  when initial() then
    reinit(x, u);   // initalize, such that der(x) = 0.
  end when
end PT1;

model Test
  PT1 b1, b2, b3;
  input u;
equation
  b1.u = u;
  connect(b1.y, b2.u);
  connect(b2.y, b3.u);
end Test;
```

Given the input signal u, all 3 blocks b1, b2, b3 are initialized at their stationary value.]

*[The **abs** and **sign** operator trigger state events if used outside of a **when** clause. If this is not desired, the **noEvent** function can be applied to them. E.g. **noEvent(abs**(v)) is |v|*

*The **div**, **rem**, **ceil**, **floor**, **integer** operators require discrete expressions as input arguments, i.e., these functions can be either called in **when** clauses or the input arguments need to be **discrete** variables. The reason for this*

restriction is that these operators are not differentiable, i.e., the partial derivatives of the result with respect to the input arguments are no continuous functions. Since this is a pre-requisite for continuous integration, it must be guaranteed that these operators are not called during continuous integration or if they are called, produce always the same result.

*The **delay** operator allows a numerical sound implementation by interpolating in the (internal) integrator polynomials, as well as a more simple realization by interpolating linearly in a buffer containing past values of expression* `expr`*. Without further information, the complete time history of the delayed signals need to be stored, because the delay time may change during simulation. To avoid excessive storage requirements and to enhance efficiency, the maximum allowed delay time has to be given via* `delayMax`*. This gives an upper bound on the values of the delayed signals which have to be stored. For realtime simulation where fixed step size integrators are used, this information is sufficient to allocate the necessary storage for the internal buffer before the simulation starts. For variable step size integrators, the buffer size is dynamic during integration. In principal, a delay operator could break algebraic loops. For simplicity, this is not supported because the minimum delay time has to be give as additional argument to be fixed at compile time. Furthermore, the maximum step size of the integrator is limited by this minimum delay time in order to avoid extrapolation in the delay buffer.]*

4 Mathematical description of Hybrid DAEs

In this section, the mapping of a Modelica model into an appropriate mathematical description form is discussed.

The result of the modeling process is a set of ordinary differential equations, often accompanied with algebraic constraint equations, thus forming a set of Differential and Algebraic Equations (DAE). The initial values of the state variables need to be specified, implying that the DAE is mathematically formulated as a so-called Initial Value Problem. This DAE is used for simulation or other analysis activities. DAEs may have discontinuities or the structure of a DAE may change at certain points in time. Such types of DAEs are called *hybrid DAE*s. Events are used to stop continuous integration at discontinuities of a hybrid DAE. After applying the discontinuous change, the integration is restarted. A hybrid DAE is mathematically described by a set of equations of the form

```
(1a) Residue Equations:  0 = f(dx/dt, x, y, t, m),  df/[dx/dt;y] is regular
(1b) Monitor Functions:  z:= g(dx/dt, x, y, t, m)
(1c) Update Equations :  0 = h(dx/dt, x^known, x^reinit, y, t, m, pre(m))
```

Additionally, every equation is a function of the parameters **p** and of the input functions **u**(t). This dependency is not explicitly shown in (1) for clarity of the equations. The variables have the following meaning:

t — time, the independent (real) variable.

x(t) — (Real) variables appearing differentiated ($\mathbf{x}^{known}$ is the part of **x** which is always known; $\mathbf{x}^{reinit}$ is the other part of **x** which is reinitialized at an event instant).

y(t) — (Real) algebraic variables.

u(t) — known (Real) functions of time.

m — discrete variables of type Real, Boolean or Integer defining the current **mode**. **pre**(**m**) are the values of **m** immediatly before the current event occured.

p — parameters, i.e., constant variables.

The *residue equations* (1a) are used for continuous integration. During integration, the discrete variables **m** are not changed. The *monitor functions* (1b) are also evaluated during continuous integration. If one of the signals **z** crosses zero, the integration is halted and an event occurs. The special case of a time event, "$z = t - t_e$", is also included. For efficiency reasons, time events are usually treated in a special way, since the time instant of such an event is known in advance. At every event instant, the *update functions* (1c) are used to determine new values of the discrete variables and of new initial values for the states **x**. The change of discrete variables may characterize a new structure of a DAE where elements of the state vector **x** are *disabled*. In other words, the number of state variables, algebraic variables and residue equations of a DAE may change at event instants by disabling the appropriate part of the DAE. For clarity of the equations, this is not explicitly shown by an additional index in (1).

At an event instant, including the initial event, the model equations are reinitialized according to the following iteration procedure:

```
loop
  solve
     0 = f(dx/dt, x^known, x^reinit, y, t, m)
     0 = h(dx/dt, x^known, x^reinit, y, t, m, pre(m))
  for dx/dt, x^reinit, y, m, where x^known, t, pre(m) are fixed
  if m == pre(m) then break
  m := pre(m)
```

```
  end loop
```

At every iteration a set of mixed Real, Boolean, Integer equations (1a),(1c) has to be solved for the indicated variables. In order that this is possible, at least the Jacobian of (1a) needs to be regular, as it was stated in (1a). This set of equations can e.g. be solved by a global fixed point iteration scheme which can be combined with the event iteration:

```
  loop
    solve "0 = f(dx/dt, x^known, x^reinit, y, t, m)"
       for dx/dt, y, where x^known, x^reinit, t, m are fixed
    solve "0 = h(dx/dt, x^known, x^reinit, y, t, m, pre(m))"
       for x^reinit, m where dx/dt, y, x^known, t, pre(m) are fixed
    if m == pre(m) then break
    m := pre(m)
  end loop
```

The hybrid DAE (1) is not the most general one, but it has a clearly defined view and structure. Especially, (1a) can be transformed into state space form, at least numerically, since the Jacobian is required to be regular. Generalizations are possible in the direction of higher index DAEs where the Jacobian of (1a) is singular. This leads to additional difficulties during integration and especially for event restart because the non-linear equation cannot be solved due to the singular Jacobian. Other generalizations concern the determination of the initial configuration by allowing the specification of any variable at the initial time and by calculating the remaining ones. A third generalization may use other algorithms to determine a consistent configuration after an event occurred, e.g., by solving a complementary problem, see (Pfeiffer and Glocker 1996) for details. For a *certain class* of higher index DAE systems, algorithms are available to automatically differentiate selected equations of (1a), choose appropriate variables to be no longer states (= dummy derivative method) and transform to a DAE (1) with a regular Jacobian.

The Modelica language allows a direct and convenient specification of physical systems. *A Modelica translator maps a Modelica model into a hybrid DAE (1),* or in one of its generalizations if these are available. The mapping into (1) is straightforward by expanding all class definitions (flattening the inheritance tree) and adding the equations and assignment statements of the expanded classes for every instance of the model to (1). The resulting hybrid DAE usually contains a huge number of sparse equations. Therefore, direct simulation of a hybrid DAE (1) which was generated by a Modelica translator requires sparse matrix methods.

There are several simulation environments available, such as Allan, Dymola, gPROMS, Ida (NMF) or Omola, which preprocess (1) symbolically to arrive at a form which can be evaluated more efficiently by numerical algorithms. Especially, efficient graph-theoretical algorithms are available to transform (1) automatically into the following form which is called **sorted** hybrid DAE:

$$
\begin{aligned}
&\text{(2a) Residue Equations:} && \mathbf{0} = \mathbf{f}^{r}(d\mathbf{x}^{i}/dt, \mathbf{y}^{i}, \mathbf{x}, t, \mathbf{m}),\\
&\text{(2b) Exp. dx-Functions:} && d\mathbf{x}^{e}/dt := \mathbf{f}^{x}(d\mathbf{x}^{i}/dt, \mathbf{y}^{i}, \mathbf{x}, t, \mathbf{m})\\
&\text{(2c) Exp. y-Functions :} && \mathbf{y}^{e} := \mathbf{f}^{y}(d\mathbf{x}^{i}/dt, \mathbf{y}^{i}, \mathbf{x}, t, \mathbf{m})\\
&\text{(2d) Monitor Functions:} && \mathbf{z} := \mathbf{g}(d\mathbf{x}^{i}/dt, \mathbf{y}^{i}, \mathbf{x}, t, \mathbf{m})\\
&\text{(2e) Update Equations :} && [\mathbf{m}, \mathbf{x}^{reinit}] := \mathbf{h}(d\mathbf{x}^{i}/dt, \mathbf{y}^{i}, \mathbf{x}^{known}, t, \mathbf{m}, \mathbf{pre}(\mathbf{m}))
\end{aligned}
$$

where the vector of algebraic variables **y** is split into implicit variables $\mathbf{y}^{i}$ and explicitly solvable algebraic variables $\mathbf{y}^{e}$. The vector of state derivatives d**x**/dt is split into implicit variables $d\mathbf{x}^{i}/dt$ and explicitly solvable variables $d\mathbf{x}^{e}/dt$, respectively. When using an *implicit integrator*, only equations (2a,2b) need to be solved during continuous integration. Equations (2c) are effectively hidden from the solver. They need only be evaluated for external usage (e.g., to store output points to be plotted). At initial time and at events, the non-linear equation of reduced dimension (2a) has to be solved. Again the dimension of the original equations has reduced considerably. It is also possible to use *explicit integration* methods, such as Runge-Kutta algorithms. During continuous integration, the integrator provides **x** and t. The model function solves (2a) for the implicit variables, uses the result to evaluate (2b) and returns the complete vector of state derivatives d**x**/dt. This procedure is useful for *real-time* simulation where only explicit one-step methods can be used and for non-stiff systems where the number of implicit equations is small and/or linear.

To summarize, a Modelica translator maps a Modelica model into the hybrid DAE (1). By a subsequent symbolic processing, (1) can be transformed into the sorted hybrid DAE (2).

5 Unit expressions

Unless otherwise stated, the syntax and semantics of unit expressions in Modelica are conform with the international standards ISO 31/0-1992 "General principles concerning quantities, units and symbols" and ISO 1000-1992 "SI units and recommendations for the use of their multiples and of certain other units". Unfortunately, neither these two standards nor other existing or emerging ISO standards define a formal syntax for unit expressions. There are recommendations and Modelica exploits them.

Examples for the syntax of unit expressions used in Modelica: "N.m", "kg.m/s2", "kg.m.s-2" "1/rad", "mm/s".

5.1 The Syntax of unit expressions

```
unit_expression:
   unit_numerator [ "/" unit_denominator ]

unit_numerator:
   "1" | unit_factors |  "(" unit_expression ")"

unit_denominator:
   unit_factor |  "(" unit_expression ")"
```

The unit of measure of a dimension free quantity is denoted by "1". The ISO standard does not define any precedence between multiplications and divisions. The ISO recommendation is to have at most one division, where the expression to the right of "/" either contains no multiplications or is enclosed within parentheses. It is also possible to use negative exponents, for example, "J/(kg.K)" may be written as "J.kg-1.K-1".

```
unit_factors:
   unit_factor [ unit_mulop  unit_factors ]

unit_mulop:
   "."
```

The ISO standard allows that a multiplication operator symbol is left out. However, Modelica enforces the ISO recommendation that each multiplication operator is explicitly written out in formal specifications. For example, Modelica does not support "Nm" for newtonmeter, but requires it to written as "N.m".

The preferred ISO symbol for the multiplication operator is a "dot" a bit above the base line: "·". Modelica supports the ISO alternative ".", which is an ordinary "dot" on the base line.

```
unit_factor:
   unit_operand [ unit_exponent ]

unit_exponent:
   [ "+" | "-" ] integer
```

The ISO standard does not define any operator symbol for exponentiation. A unit_factor consists of a unit_operand possibly suffixed by a possibly signed integer number, which is interpreted as an exponent. There must be no spacing between the unit_operand and a possible unit_exponent.

```
unit_operand:
   unit_symbol | unit_prefix unit_symbol

unit_prefix:
  Y | Z | E | P | T | G | M | k | h | da | d | c | m | u | p | f | a | z |
y
```

A unit_symbol is a string of letters. A basic support of units in Modelica should know the basic and derived units of the SI system. It is possible to support user defined unit symbols. In the base version Greek letters is not supported, but full names must then be written, for example "Ohm".

A unit_operand should first be interpreted as a unit_symbol and only if not successful the second alternative assuming a prefixed operand should be exploited. There must be no spacing between the unit_symbol and a possible unit_prefix. The value of the prefixes are according to the ISO standard. The letter "u" is used as a symbol for the prefix micro.

5.2 Examples

- The unit expression "m" means meter and not milli (10^{-3}), since prefixes cannot be used in isolation. For millimeter use "mm" and for squaremeter, m^2, write "m2".
- The expression "mm2" means $mm^2 = (10^{-3}m)^2 = 10^{-6}m^2$. Note that exponentiation includes the prefix.

  The unit expression "T" means Tesla, but note that the letter "T" is also the symbol for the prefix tera which has a multiplier value of 10^{12}.

6 External function interface

6.1 Overview

The purpose is to allow calls to functions defined outside of the Modelica language. The design goals were:

- C functions are used as the least common denominator. It is planned that other languages (C++, Fortran) will be supported in the future too.
- A mapping of argument types from Modelica to the target language should be defined.
- The mapping should be "natural" in the sense that there is a mapping from Modelica to standard libraries of the target language.

It should be possible to specify inverse and Jacobian functions. Details will be specified in a future Modelica release.

The format of an external function declaration is as follows.

```
function IDENT
  [ input-declarations ]
  [ output-declaration ]
  external

end IDENT;
```

Examples of Modelica function declarations:

```
function log
  input  Real x;
  output Real y;
  external
end log;
function PolynomialEvaluator2
  input  Real a[:];
  input  Real x;
  output Real y;
  external
end PolynomialEvaluator2;
function Force3D
  input     Real c=1.0;
  input     Real s;
  output    Real f[3];
  external
end Force3D;
```

The corresponding declarations in the C language are:

```
extern double log(double x);
extern double PolynomialEvaluator2(double t* a, size_t dim1, double x);
extern void   Force3D(double c, double s, double *f, size_t dim1);
```

6.2 Mapping of argument types

The arguments of the external function are declared in the same order as in the Modelica declaration. The single Modelica function output parameter specifies the return type of the external function.

6.2.1 Simple types

Arguments of **simple** types are by default mapped as follows:

Modelica	C	
	Input	Output (return value)
Real	`double`	`double`
Integer	`int`	`int`
Boolean	`int`	`int`
String	`const char *`	`const char *`

Strings are nul-terminated to facilitate calling of C functions.

6.2.2 Arrays

Arrays of simple types are mapped to an argument of the simple type, followed by n arguments of type `size_t` with the corresponding array dimensions. The type `size_t` is a C unsigned integer type. Storage for arrays as return values is allocated by the calling routine, so the dimensions of the returned array is fixed.

Arrays are stored in column-major order according to the Fortran conventions, in order to be compatible with most standard numerical libraries.

Modelica	C
	Input and output
T[*dim1*]	*T* *, `size_t` *dim1*
T[*dim1*, *dim2*]	*T* *, `size_t` *dim1*, `size_t` *dim2*
T[...]	*T* *, `size_t` *dim1*, ..., `size_t` *dimn*

6.2.3 Records

A Modelica record class that contains simple types, other record elements, or arrays with fixed dimensions thereof, are mapped as follows:

- The record class is represented by a struct in C.
- Each element of the Modelica record is mapped to its corresponding C representation.
  The elements of the Modelica record class are declared in the same order in the C struct.
- Arrays are mapped to the corresponding C array.

For example,

```
record R                                    struct R {
  Real x;                                     double x;
  Integer y[10];       is mapped to           int    y[10];
  Real z;                                     double z;
end R;                                      };
```

7 Modelica standard library

The pre-defined, free "**package** Modelica" is shipped together with a Modelica translator. It is an extensive standard library of pre-defined components in several domains. Futhermore, it contains a standard set of **type** and **interface** definitions in order to influence the trivial decisions of model design process. If, as far as possible, standard quantity types and connectors are relied on in modeling work, model compatibility and thereby reuse is enhanced. Achieving model compatibility, without having to resort to explicit coordination of modeling activities, is essential to the formation of globally accessible libraries. Naturally, a modeller is not required to use the standard library and may also add any number of local base definitions.

The library will be amended and revised as part of the ordinary language revision process. It is expected that informal standard base classes will develop in various domains and that these gradually will be incorporated into the Modelica standard library.

The type definitions in the library are based on ISO 31-1992. Several ISO quantities have long names that tend to become awkward in practical modeling work. For this reason, shorter alias-names are also provided if necessary. Using, e.g., "ElectricPotential" repeatedly in a model becomes cumbersome and therefore "Voltage" is also supplied as an alternative.

The standard library is not limited to pure SI units. Whenever common engineering practice uses a different set of (possibly inconsistent) units, corresponding quantities will be allowed in the standard library, for example English units. It is also frequently common to write models with respect to scaled SI units in order to improve the condition of the model equations or to keep the actual values around one for easier reading and writing of numbers.

The connectors and partial models have predefined graphical attributes in order that the basic visual appearance is the same in all Modelica based systems.

The complete Modelica package can be downloaded. Below, the introductory documentation of this library is given together with links to the subpackages. Note, that the Modelica package is still under development.

package Modelica
  package Info
    /* The Modelica package is a standardized, pre-defined and **free**
       package, that is shipped together with a Modelica translator. The
       package provides constants, types, connectors, partial models and
       model components in various disciplines.

       In the Modelica package the following conventions are used:

       - Class and instance names are written in upper and lower case
         letters, e.g., "ElectricCurrent". An underscore is only used
         at the end of a name to characterize a lower or upper index,
         e.g., body_low_up.

       - Type names start always with an upper case letter.
         Instance names start always with a lower case letter with only
         a few exceptions, such as "T" for a temperature instance.

       - A package XXX has its interface definitions in subpackage
         XXX.Interface, e.g., Electric.Interface.

       - Preferred instance names for connectors:

```
    p,n: positive and negative side of a partial model.
    a,b: side "a" and side "b" of a partial model
       (= connectors are completely equivalent).

  The following subpackages are available:

  GENERAL PACKAGES
    Constant          Mathematical and physical constants
    Math              Mathematical functions
    SIunit            SI-unit type definitions

  FORMALISM PACKAGES
    BlockDiagram      Input/output blocks
    BondGraph         Bond graph components
    FiniteStateMachine  Finite state machine
    PetriNet          One-token petri-nets.

  GENERAL DOMAINS
    Electric          Electric and electronic components
    Mechanics         1D and 3D mechanical components
    ThermoFluid       1D thermo-fluid components

  DOMAIN PACKAGES
    Aircraft          Aircraft components
    Building          Energy balance of building components
    DriveTrain        Planetary gearboxes, clutches
    ElectricPower     Generators, motors, electric line
    Hydraulics        Hydraulic components
  */
 end Info;
end Modelica;
```